# Octagon Lesson Plan

### Concept/principle to be demonstrated:

The isosceles right triangle provides a math constant used to solve octagon problems. Octagons can be drawn from either a circle or square. The unique properties of the isosceles right triangle provide the mathematical answer when various dimensions are given. The simplest way to demonstrated understanding is by applying the constant to solve additional construction related problems using a calculator.

### Lesson objectives/Evidence of Learning:

- Comprehends concept of octagon layout
- Uses physical, symbolic, and technological models to explore conjectures
- Uses basic 2-D figures such as circles and polygons to represent objects essential to a situatio
- Introduces a coordinate system when useful for describing the position of objects in a situation
- Calculates the area and perimeter of circles, triangles, quadrilaterals, and regular polygons
- Applies formula to solve variety of construction problems
- Uses calculator to compute accurately

### How this math connects to construction jobs:

Octagons provide a variety of special effects or features to a building being constructed. Sometimes only a few sides of the octagon are used. This lesson will help students comprehend how octagons are used by many trades.

- **Carpenters** use three sides of an octagon to build walls for a bay window.
- Gazebos are typically made in an octagon shape providing challenges to **framers and roofers**.
- **Metal stud framers** use formulas to build octagon shaped rooms and openings in walls.
- **Sheet metal workers** use octagon air diffusers.

### Teacher used training aids:

- 8 1/2” x 11” paper printed with 7” square (Or use marked plywood square)
- Length of tie wire (or other soft wire) bent to fit isosceles triangle in corner of square
- Additional applications are found on 45°-45°-90° Triangles handout if needed.

### Materials needed per student:

- 8 1/2” x 11” paper printed with 7” square
- 6-8” piece of string
- Pencil and 6” straight edge (any book will work)
- Calculator with √ key & memory +/- functions
- ***Octagon Worksheet***
- ***Octagon Inscribed in a Square*** handout
- ***Special Right Triangles*** handouts

## Terms:

- **Hypotenuse:** The side opposite the right angle in a right triangle.
- **Isosceles right triangle:** A triangle with at least two equal sides.
- **Octagon: A**n eight sided shape with all side and angles equal.
- **Right triangle:** A triangle that contains a right angle (90 degree measurement)
- **Square ($^2$):** A quadrilateral with four equal sides and four 90 degree angles.
- **Square root (√):** The square root of x is the number that, when multiplied by itself, gives the number, x.

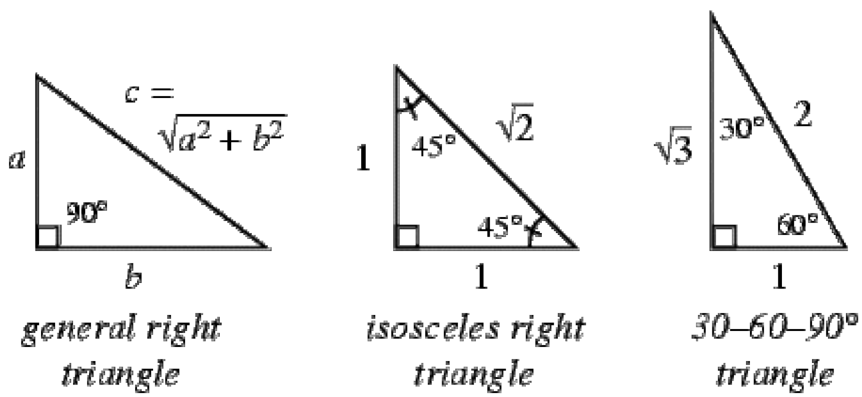
## Lesson Introduction:

The octagon is probably the most used geometrical figure in building. Often in layout work one of several formulas is used to find the length of a side.

## Lesson Components:

1. Right triangles are special:
   a. Used extensively in construction.
   b. 45°- 45°-90° and 30°- 60°- 90° have unique qualities.

2. Draw on white board and explain:

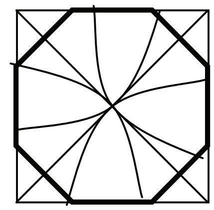



3. Isosceles right triangle has legs of the same length and 45° angles. An adaptation to the Pythagorean Theorem is useful.
   a. $a^2 + b^2 = c^2$
   b. The ratio of the legs to the hypotenuse is 1: √2 because:
   c. $1^2 + 1^2 = c^2$ $1 + 1 = 2$ $c^2 = 2$ $c = \sqrt{2}$
   d. a and b are equal therefore:
   e. $2a^2 = c^2$
   f. $(2)(1)^2 = (2)(1) = 2$ again $c^2 = 2$ and $c = \sqrt{2}$

4. Many times in construction, an octagon is drawn based on the dimensions of another shape. It may be inside (inscribed) either a square or circle. Other times the octagon is drawn outside a circle (called either described or super scribed).

5. Today’s activities will be based inside a square.

6. **Demonstration** using isosceles right triangle to lay out an octagon:
   a) Craft workers lay out octagons by making a series of arches and lines.
   b) Draw square on board and add diagonal lines on square or use paper/plywood model.
   c) Swing arches to lay out octagon.
   d) Connect octagon sides as shown:

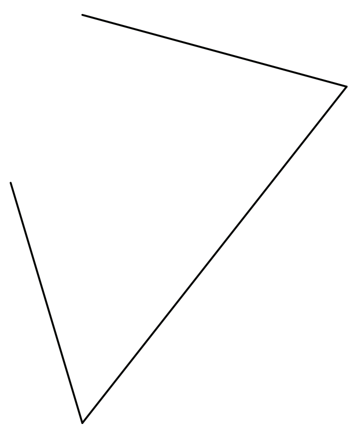


   e) Have students fold diagonals of printed 7” square or use straight edge to mark.
      i. Tie a loop in the string to fit pencil
      ii. Students swing arches holding string in corner with finger
      iii. Use straight edge to connect octagon sides

   f) Show students the isosceles right triangle formed in each corner
   g) Tell students that applying the special formulas for isosceles right triangles
   h) Review octagon formulas.(pass out Octagon inscribed in a square)
      i. Side of square ÷ 2.414 = length of octagon side
      ii. Side of square ÷ 3.414 = distance from corner to octagon side
      iii. Side of square x square root of 2 – side of square = side of octagon

   i) Ask students why these formulas work
      i. √2=1.414
      i. Each of these formulas uses √2
      iii. 1.414 + 1 = 2.414
      iv. 1 + 1 + 1.414 = 3.414

   j) Bend wire to fit a triangle into a corner (note: hypotenuse should be in middle)



   k) Straighten wire to show that its length equals the side of the square.

7. Help students to complete worksheet.

Name ______________________________

# Octagon Worksheet

### Problem #1

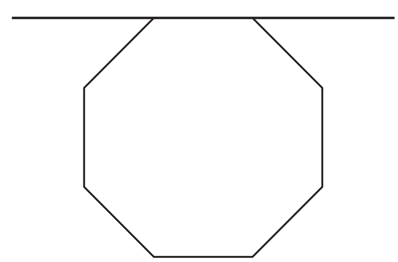
Tile setters are installing an octagon feature in the center of a floor in a square room. The walls are 12 feet long. What is the size of the octagon side? (round answer to nearest 10th of a foot.)

### Problem #2

How far from the corner is the octagon in Problem #1? Give the answer in feet and inches. (round answer to nearest 1/16th of an inch).

### Problem #3

How many degrees are in each angle of the octagon?



### Problem #4

What is the length of the square needed to install an octagon stained glass piece with 16” sides? (round answer to the nearest 1/16th of an inch.)

**KEY**

# Octagon Worksheet

**Problem #1**

Tile setters are installing an octagon feature in the center of a floor in a square room. The walls are 12 feet long. What is the size of the octagon side? (round answer to nearest 10th of a foot.)

**Side of square ÷ 2.414 = length of octagon side**
**12’ ÷ 2.414 = length of octagon side = 4.97’ = 4’-11 5/8’**

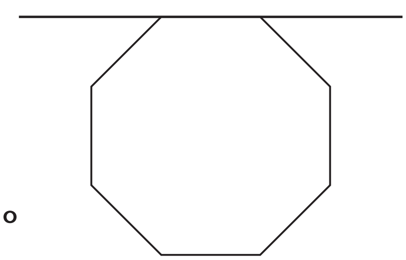
**Problem #2**

How far from the corner is the octagon in Problem #1? Give the answer in feet and inches. (round answer to nearest 1/16th of an inch).

**Side of square ÷ 3.414 = distance from corner to octagon side**
**12’ ÷ 3.414 = distance from corner to octagon side = 3.515’ = 3’-6 3/16”**
**OR 12’ minus 4’-11 5/8” = 7’-3/8” divided by 2 = 3’-6 3/16”**

**Problem #3**

How many degrees are in each angle of the octagon?



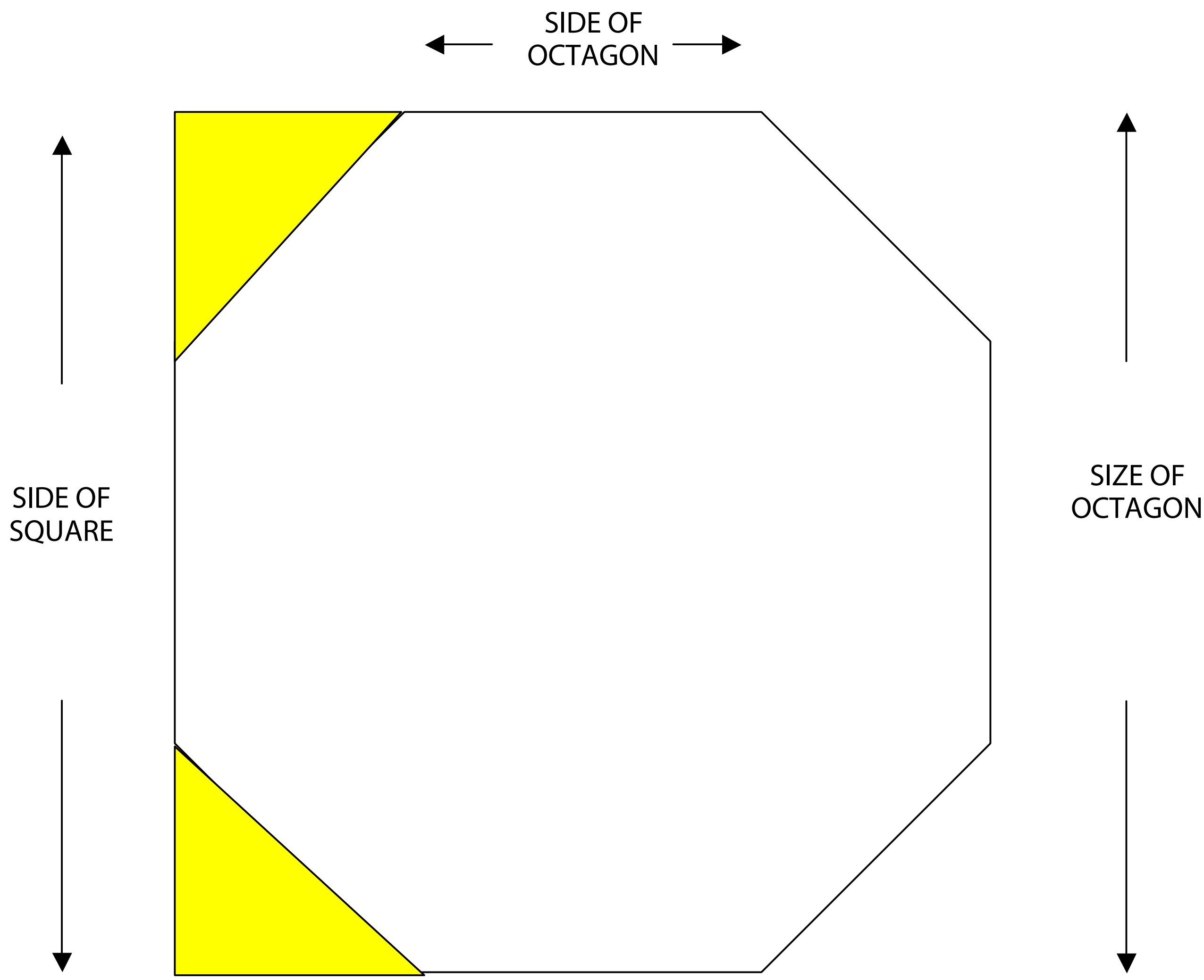
**Straight line = 180°**
**Angle of isosceles right triangle is 45°**
**180° - 45° = 135°**

**Problem #4**

What is the length of the square needed to install an octagon stained glass piece with 16” sides? (round answer to the nearest 1/16th of an inch.)

**16” x 2.414 = 38.624 = 38 5/8”**

# Octagon Inscribed in a Square




Size of octagon and side of square are equal

## Formulas

1. Side of square ÷ 2.414 = length of octagon side
2. Side of square ÷ 3.414 = distance from corner to octagon side
3. Side of square x square root of 2 – side of square = side of octagon

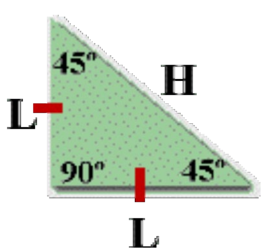
# Special Right Triangles
## $45^{\circ}$-$45^{\circ}$-$90^{\circ}$

Certain triangles possess "special" properties that allow us to use "short cut formulas" in arriving at information about their measures. These formulas let us arrive at the answer very quickly.

One such triangle is the $45^{\circ}$-$45^{\circ}$-$90^{\circ}$ triangle.

There are two "special" formulas that apply **ONLY** to the $45^{\circ}$-$45^{\circ}$-$90^{\circ}$ triangle.

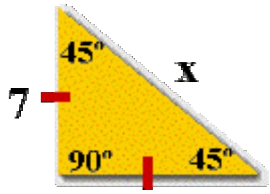
## $45^{\circ}$-$45^{\circ}$-$90^{\circ}$ (Isosceles Right Triangle) "Special" Formulas

| | | | |
|---|---|---|---|
|   | $H = L\sqrt{2}$ | $L = \frac{1}{2}H\sqrt{2}$ | **You must remember that these formulas can be used ONLY in a $45^{\circ}$-$45^{\circ}$-$90^{\circ}$ triangle.** |

## What should I do if I forget the formulas?

The nice thing about mathematics is that there is always another way to do the problem. If you forget these formulas, you could always use the Pythagorean Theorem or a Trigonometry formula.

Let's look at 3 solutions to this problem where you are asked to **find x**:




| Special Formula solution | Pythagorean Theorem solution | Trigonometric solution |
|---|---|---|
| We are looking for the hypotenuse so we will use the formula that will give the answer for the hypotenuse:<br>$\mathbf{H} = \mathbf{L}\sqrt{2}$<br>Substituting the leg = 7, we arrive at the answer:<br>$\mathbf{x} = 7\sqrt{2}$<br>A nice feature of these special formulas is that the answer is already in reduced form. | Since a 45º-45º-90º, also called an isosceles right triangle, has two legs equal, we know that the other leg also has a length of 7 units.<br>c2 = a2 + b2<br>x2 = 72 +72<br>x2 = 49 + 49<br>x2 = 98<br>$\mathbf{x} = \sqrt{98}$<br>$\mathbf{x} = \sqrt{49} \bullet \sqrt{2}$<br>$\mathbf{x} = 7\sqrt{2}$ | Use either 45º angle as the reference angle (where your stick figure will stand). One possible solution is shown below:<br>$\sin\ 45^\circ = \dfrac{7}{x}$<br>$.7071 = \dfrac{7}{x}$<br>$\mathbf{x}$=**9.9** rounded<br>($7\sqrt{2}$=**9.9**) rounded |