Laying the Foundation:
CONSTRUCTION MATH

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Construction Center of Excellence
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Laying the Foundation:
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Laying the Foundation: CONSTRUCTION MATH

Why We Created This Booklet

We created this booklet in response to college instructors and apprenticeship coordinators who are concerned about the math skills of their incoming students. According to the state's Transition Math Project, 50% of students who enter Washington State's community and technical colleges directly after high school graduation need to take developmental math classes before they are ready for college-level math.[1] Furthermore, many students entering professional-technical programs (including apprenticeship programs) have been out of high school for several years before they enroll, suggesting that their math skills may be even rustier than their younger colleagues.

There are probably many reasons for the inadequate math preparation of so many entering college students. We believe that one key reason, at least in the construction trades, is that high school students – and some of the teachers and counselors that advise them – do not realize the extent to which construction workers must use their minds as well as their hands. Therefore, many high school students do not take the math classes that would help them in their chosen career field, or they tune out in class because they do not understand the relevance of their math lessons.

We hope that, by providing examples of the math used in college-level construction classes, we will give high school teachers and counselors the tools to make math more relevant to students, and ultimately produce students who are better-prepared for further education in their chosen career field. This booklet can be used by high school math and construction teachers for in-class activities or homework assignments. It can also be used by teachers and counselors as an advising tool for students who want to know "why do I have to learn this?"

What Kind of Construction? What Kind of Math?

When most people think of construction, they probably think of carpenters, plumbers, electricians, and painters: people who are involved in building a structure. These jobs are certainly important, as well as being some of the most highly visible and plentiful jobs in the construction industry. However, the life cycle of a building starts long before the foundation is poured and continues long after the occupants have moved in. The construction industry also includes drafters, engineers, and architects who design a building and create blueprints; surveyors who measure the site and verify boundaries for the property; project managers who estimate the cost of building materials and personnel; and

[1] The Transition Math Project is an initiative sponsored by the State Board of Community and Technical Colleges, the Office of Superintendent of Public Instruction, Higher Education Coordinating Board, and Council of Presidents. For more information on the Transition Math Project, see www.transitionmathproject.org.

engineers who maintain the building's mechanical and electrical systems once the construction is complete. This booklet includes examples from a wide variety of career training programs in the construction industry.

The examples in this booklet include general arithmetic (involving whole numbers as well as fractions), algebra, and geometry. Some fields, such as surveying, use trigonometry and calculus.

Construction Math Can Help with the WASL!

High school teachers today often feel pressured by the need to prepare their students for high-stakes testing such as the Washington Assessment of Student Learning (WASL). Some school districts may encourage the reduction or elimination of any subject matter that is not clearly related to the WASL. However, we believe that, by helping students connect the math that they learn to their ultimate career goals, they are more likely to pay attention, retain the information, and ultimately perform better on standardized tests. Also, the math in this book will help students accomplish Goal 4 of the state's Essential Academic Learning Requirements: Understand the importance of work and how performance, effort, and decisions directly affect future career and educational opportunities.

Resources for Construction Math
Websites, Videos, and books

Websites:
Math for Construction Trades 2002
www.vbisd.org/resources/curriculum/math

In on the Ground Floor
How is math used in construction
www.creativille.org/groundfloor/index.htm

Math to Build On
http://mathforum.org/~sarah/hamilton/math.build.html

Construction Math 101
http://doityourself.com/build/construction.htm

Vocational Information Center:
Math Resources - Tutorials, Formulas, and Directories
http://www.khake.com/page47.html

Vocational Information Center:
Construction and Trade Lesson Plans
http://www.khake.com/page82.html

Videos:
Math for the Construction Trades, Prentice Hall Media, Inc.

Math at Work, producer Jocelyn Riley
Women in nontraditional careers

Books:
Workshop Math
Math for construction, workshop and the home
Robert Scharff

Math to Build On
A book for those who build
Johnny E. Hamilton and Margaret S. Hamilton

Pipe Fitters and Pipe Welders Handbook
Thomas W. Frankland

Pipe Fitter's Math Guide
Johnny E. Hamilton

Carpentry Math
David Wiltsie

Contemporary's Number Power 2 : Fractions, Decimals, and Percents
Jerry Howett

COORDINATE GEOMETRY
(COGO)

To find the coordinates of a point, given an Azimuth (or a Bearing) from a known point to the unknown point, and a distance between a known point and the unknown point.

When using Azimuths:

The Sine of the Azimuth multiplied by the distance = Δ East
The cosine of the Azimuth multiplied by the distance = Δ North
After the Δ North and Δ East are calculated, simply **ADD** the values to the coordinates of the known point.

Note:
When using the Azimuth to calculate the Δ North or the Δ East you will often calculate a negative number. When this happens you **MUST** respect the sigh when adding the values to the coordinates of the known point.

Example:
Given:
Coordinates for the known point are N10,000.00, E5,000.00
The Azimuth from known point to unknown point is 303°46'25"
The distance from the known point to the unknown point is 597.36'
Calculate:
Sine 303°46'25" x 597.36' = -496.55'
Cosine 303°46'25" x 597.36' = 332.08'
Northing of unknown point = 10,000.00' + 332.08' = 10,332.08'
Easting of unknown point = 5,000.00' + (-496.55) = 4,503.45'

When using Bearings:

When calculating coordinates using bearings you must pay attention to the direction of the bearing.
If you have a North bearing, you must **ADD** the Δ North to the North coordinate of the known point.
If you have a South bearing, you must **SUBTRACT** the Δ North from the North coordinate of the known point.
If you have a East bearing, you must **ADD** the Δ East to the East coordinate of the known point.
If you have a West bearing, you must **SUBTRACT** the Δ East from the East coordinate of the known point.

COORDINATE GEOMETRY
(COGO)

Example:

Given:

The coordinates of the known point = N10, 000.00', E5, 000.00'
The bearing from the known point to the unknown point is N 56°13'35"W
The distance from the known point to the unknown point is 597.36'

Calculate:

Sine 56°13'35" x 597.36' = 496.55'
Cosine 56°13'35" x 597.36' = 332.08'

Northing of unknown point = 10,000.00' + 332.08' = 10,332.08'
Easting of unknown point = 5,000.00' - 496.55 = 4,503.45'

Note:
Many people believe it is easier to use Azimuths instead of Bearings to calculate coordinates because you always add your calculation to the known coordinate. In order to do this you must understand how to convert Bearings to Azimuths. You must also understand how to calculate Azimuths to Bearings. To do this, the following rules apply.

Calculating Azimuths from Bearings

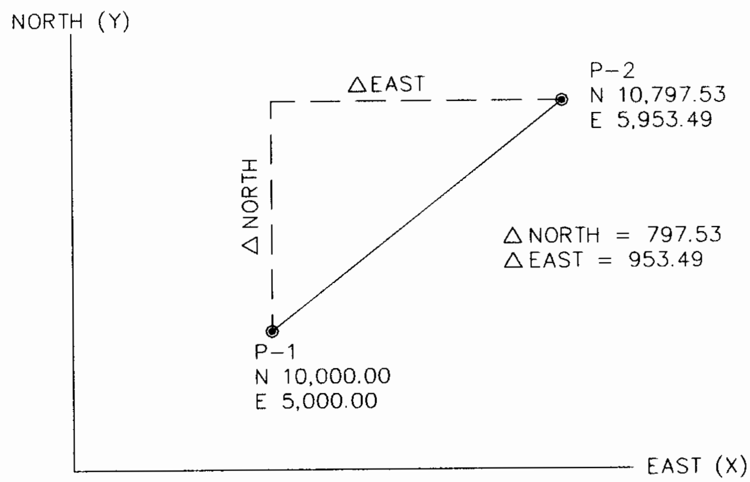
1. In the Northeast quadrant, the Azimuth and Bearing are the same.
2. In the Southeast quadrant, you must subtract the Bearing from 180° to get the Azimuth.
3. In the Southwest quadrant, you must add the bearing to 180° to get the Azimuth.
4. In the Northwest quadrant, you must subtract the Bearing from 360° to get the Azimuth.

Calculating Bearings from Azimuths

1. Between 0° and 90° the Bearing and Azimuth are the same.
2. Between 90° and 180° you must subtract the Azimuth from 180° to get the Bearing.
3. Between 180° and 270° you must subtract 180° from the Azimuth to get the Bearing.
4. Between 270° and 360° you must subtract the Azimuth from 360° to get the Bearing.

COORDINATE GEOMETRY
(COGO)

The science of coordinate geometry states that if two perpendicular directions are known such as an X and Y plane (North and East). The location of any point can be found with respect to the origin of the coordinate system, or with respect to some other known point on the coordinate system. This is accomplished by finding the deference between the X and Y coordinates (North and East) of known and unknown point and adding that deference to the known point. The magnitude and direction (Azimuth and distance) can also be found between two points if the coordinates of the two points are known.
The rules of Coordinate Geometry are basic and are as follows:



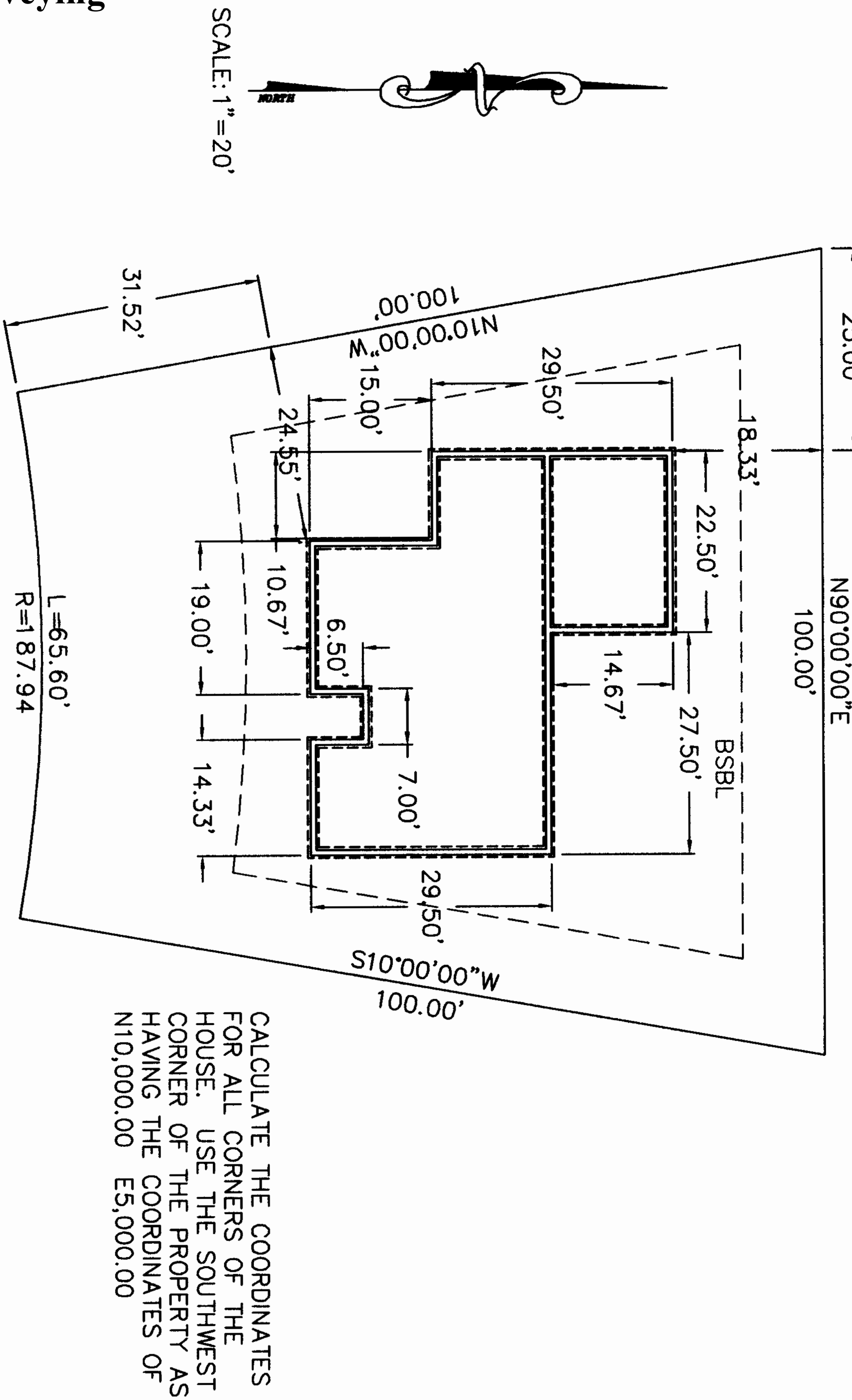
Inversing:

The Azimuth from P-1 to P-2 = $Tan^{-1}\left(\frac{\Delta East}{\Delta North}\right)$

Note:
Upon completion of the above calculation your answer will be in degrees and decimals of a degree. You **must** convert to degrees, minutes, and seconds.

The distance from P-1 to P-2 = $\sqrt{(\Delta North)^2 + (\Delta East)^2}$

Surveying



SCALE: 1" =20'
NORTH
31.52'
100.00'
N10°00'00"W
25.00'
29.50'
15.00'
24.55'
18.33'
22.50'
10.67'
19.00'
6.50'
14.67'
27.50'
BSBL
N90°00'00"E
100.00'
L=65.60'
R=187.94
7.00'
14.33'
29.50'
S10°00'00"W
100.00'
CALCULATE THE COORDINATES FOR ALL CORNERS OF THE HOUSE. USE THE SOUTHWEST CORNER OF THE PROPERTY AS HAVING THE COORDINATES OF N10,000.00 E5,000.00

Soil Excavation

Introduction

In this module, we are going to use the process of excavating soil to demonstrate the mathematical concept of percent changes and problem solving skills. When digging a hole in your backyard or at a construction site, you may have noticed that the pile of soil is larger than the hole that you dug. This is a natural occurrence; loose stock-piled soil takes up more volume than compact soil.

Knowing how much the volume changes is necessary so that correct estimates of the hauling costs can be made. If 5 cubic yards of soil need to be excavated from a bank, then the site manager better plan on hauling and dumping more than 5 cubic yards. Fortunately, the swellage and shrinkage of soil follows a predictable pattern. There are charts that can be used to determine the amount of change in the volume of soil.

In order to calculate the volumes of loose or compact soil, we'll be looking at percentage changes. In Chapter 3 of the textbook, you learned the basic formula $P = B \times R$ to solve percentage problems. With this formula, you used a two step process for determining the ending values. When dealing with percent increases (decreases), you determined the part, also called the amount of change. Then, you added (subtracted) to the base, also called the beginning value. In this module, we'll study an alternative method that matches how you will commonly be given information in charts that will encounter in your work. Also, we'll study a systematic approach to calculate percent changes using information from different types of charts.

Percent Increase/Decrease

Example #1: When excavating compact sand, the volume of material will actually increase by approximately 15% when stock-piled or loaded on a truck. So, if you were to dig a hole that has a volume of 2 cubic yards, you would have to haul away more than 2 cubic yards. In order to budget the job correctly, we'll need to know the volume of soil that's being hauled away and dumped. If we have 2 cubic yards of compact sand, what's the volume of loose, stock-piled sand?

To answer a question about the volume of the loose, stock-piled sand, let's look at two approaches. The first approach was presented in Section 3.5 of the textbook and the second approach will be presented in this module. The two approaches are closely related. Both approaches have two steps and deal with multiplication and addition (subtraction).

Approach #1 (Calculating the Change). Let's review the approach presented in Section 3.5 of the textbook and uses the formula $P = B \times R$. This formula is used to calculate the amount of change when given a percentage rate. The letter P stands for the amount of change and is also called the part. The letter B stands for the beginning amount and is also called the base. The letter R stands for the percentage rate and is calculated using the decimal version of the percent. If we have a percent change of 15%, then we calculate using 0.15. In other words, to calculate the percent change, we move the decimal two places.

For our soil situation, P is the amount of increase in volume, B is the volume of the compacted soil, and R is the percent rate of change. We can find the change in volume by calculating

$$P = 2 \times 0.15 = 0.3 \text{ cubic yards.}$$

Then, we add the 0.3 to the original volume of 2 to get 2.3. Thus, 2 cubic yards of compact sand dug from the ground will become 2.3 cubic yards of loose, stock-piled sand that needs to be hauled away. We get 2.3 cubic yards by adding together the original volume and the change in volume obtained using the percent.

Approach #2 (Combining the Percents). Now, let's look at another way of calculating the volume of loose, stock-piled sand that needs to be hauled away. If the volume is going to increase by 15%, then the amount to be hauled away is going to be 115% of the volume of the compact sand. We get 115% by adding 100% and 15%; the original size of the hole plus the extra volume that we get for being loose sand.

Then, we convert the 115% into its decimal version, just like we did with the 15%. The decimal version of 115% is 1.15. The volume of loose, stock-piled sand is calculated by

$$2 \times 1.15 = 2.3 \text{ cubic yards.}$$

Notice, this result is the same answer that we got from using the method presented in Section 3.5 of the textbook.

Let's take a moment to recap the two approaches for calculating percent increases. Approach #1 was the method taught in Section 3.5. In the first step, we calculated the change by multiplying the beginning volume by the percent change. In the second step, we added the amount of change to the original value to get the new total value. In Approach #2, we added the percent change to 100%. Then, we multiplied the beginning volume by this new percentage. Both approaches used multiplication and addition, but in a different order. Chart 1.1 shows the steps of the two approaches along with the example that was used above.

Chart 1.1: Approaches for Percent Increases		
Approach	Steps	Example
#1: Using the change in value. (Section 3.5)	1. Use the percent to find the change. 2. Add the change to the original value.	1. $0.15 \times 2 = 0.3$ 2. $2 + 0.3 = 2.3$
#2: Using the combined percent.	1. Add the percent to 100%. 2. Use the new percent to find the new total value.	1. $15\% + 100\% = 115\%$ 2. $1.15 \times 2 = 2.3$

Example #2: Let's look at another example. Suppose we need to excavate 6 cubic yards of clay. In order to determine the cost of the excavation, we'll need to figure out the volume of the loose clay. The volume of loose clay is approximately 35% larger than compact clay. What is the volume of loose clay when we start with 6 cubic yards of compact clay?

Using Approach #1, we calculate the change in the volume of compact soil to loose soil: $6 \times 0.35 = 2.1$. Then, we add the change in volume to the original volume: $6 + 2.1 = 8.1$. Thus, 6 cubic yards of compact clay will become 8.1 cubic yards of loose clay.

Using Approach #2, we add the percents: $100\% + 35\% = 135\%$. Then, we multiply to get the new volume: $6 \times 1.35 = 8.1$. We get the same answer of 8.1 cubic yards. Chart 1.2 shows the steps for the two methods.

Chart 1.2: Approaches for Percent Increases		
Approach	Steps	Example
#1: Using the change in value. (Section 3.5)	1. Use the percent to find the change. 2. Add the change to the original value.	1. $6 \times 0.35 = 2.1$ 2. $6 + 2.1 = 8.1$
#2: Using the combined percent.	1. Add the percent to 100%. 2. Use the new percent to find the new total value.	1. $100\% + 35\% = 135\%$ 2. $6 \times 1.35 = 8.1$

Multipliers

At this point, let's bring in a new math term. When calculating with percents, we change the percent to the decimal version. Then, we multiply. The decimal version of the percent is called a **multiplier**. In Example #1, we multiplied 2 by 1.15. Let's call the 1.15 a multiplier. In Example #2, we multiplied 6 by 1.35. Let's call the 1.35 a multiplier. In terms of formulas like $A = k \times B$, the k is the multiplier. The *k*-value is the factor needed to go from B to A. In the formula $P = B \times R$ found in the textbook, R is the multiplier because it was the decimal version of the percent. And, it relates *base* and the *part*.

Key Mathematical Relationship of Multipliers

When something increases in size based on a percent change, the multiplier has to be larger than 1. When multiplying, the only way to go from a small number to a bigger number is when the multiplier is larger than 1. On the other hand, when something decreases in size, the multiplier has to be between 0 and 1. (Note: a multiplier of 1 doesn't change anything. You get the same value back as you started with when multiplying by 1. That's why 1 is the boundary between increasing and decreasing in value. Additionally, the multiplier of 1 corresponds to 100%.)

When we have a 25% increase, then we have the multiplier 1.25. The multiplier is larger than 1 because we are increasing in value by 25%. We're adding 25% and 100%, which will be larger than 100%. Now, if we have a 25% *decrease*, then we subtract 25% from 100% to get 75%. If we have a 25% decrease, then we'll end up with only 75% of the value with what we started. And, the decimal version of 75% is 0.75. Thus, when decreasing by 25%, the multiplier is 0.75. Notice, by subtracting the percent change, we'll always get a multiplier that's less than 1, which ends up with a smaller value.

Check Your Understanding

A1. For the following percent changes, determine the multiplier:

(a) +37%   (b) −32%   (c) +2.8%   (d) −14%

A2. For the following multipliers, determine the percent change:

(a) 1.42   (b) 0.68   (c) 0.84   (d) 1.016

Answers: A1(a) 1.37, (b) 0.68, (c) 1.028, (d) 0.86. A2(a) +42%, (b) -32%, (c) -16%, (d) +1.6%.

Application of Multipliers

Now, you might be thinking why do I need Approach #2 for calculating percent changes? You might ask, when would we even use it? So far, you've been able to use Approach #1 without any problems and was useful because we where given the percent change. However, there are charts that you will encounter in your future classes and in your work that don't provide the percent change. The information in Chart 1.3 provides the multipliers that are used to calculate the volume of excavated and loose

Chart 1.3: Approximate Swellage Factors for Various Soil Types (calculated from 100% volume in place to excavated and loose material ready for transport.)

Soil Type	100% relative compaction (factor)	Excavated and loose (factor)
Sand	1	1.15
Sandy-Loam	1	1.20
Clay-Loam	1	1.30
Clay	1	1.35

soil when you know the volume of the hole or bank. The values given in the third column are multipliers which are the factor needed to go from the volume of compact soil to the volume of excavated and loose soil. Multiplier is just another word for a factor.

You may have noticed that we've already used the information about both Sand and Clay soils in the previous examples. Let's take a moment to look at the mathematical relationships we've talked about so far. In Example 1, the volume of the sand increased by 15% and we determined that the multiplier was 1.15. In Chart 1.3, look at the number in the third column of the Sand line. We see the factor of 1.15, the multiplier that includes the 15% increase in size. Now, take a look at the factor for clay. We see our multiplier of 1.35, which indicates a 35% increase in volume. The chart provides the multipliers for four soil types.

From Chart 1.3, we can determine the percent change in the volume of a sandy-loam soil. The factor is 1.20, which corresponds to 120%. The volume of loose sandy-loam soil is 20% bigger then when it's compact, because 120% is 20% more than 100%.

Check Your Understanding

B1. What is the percent change in volume for Clay-Loam soil to go from compact soil to loose soil?

B2. If 12 cubic yards of clay soil needs to be excavated, what is the volume of the loose clay soil that needs to be hauled away?

B3. Suppose it costs $5 per cubic yard to dump sandy-loam soil at a different site. If 22 cubic yards is to be excavated, what is the cost of dumping the soil? (Note: to keep it simple, we're using just the dumping fees not the costs of excavating and hauling.)

B4. Suppose 18 cubic yards of sandy-gravel is to be hauled away and the volume of excavated and loose sandy-gravel increases in volume by 42%. What is the volume of loose sandy-gravel that is hauled away?

B5. What size does the multiplier have to be so that there is an increase in volume?

Answers: 1. 30%. 2. 16.2 cubic yards. 3. $132. 4. 25.6 cubic yards. 5. Larger than 1.

Chart 1.4: Approximate Shrinkage Factors for Various Soil Types (calculated from backfill material in a loose, friable state at a time of use and subsequent normal setting.)		
Soil Type	Loose-stockpiled material	Normal shrinkage
Sand	1	*1.18
Sandy-Loam	1	1.23
Clay-Loam	1	1.43
Clay	1	1.48
*Explanation: shrinkage factor must be larger than swellage factor to provide for the shrinkage of that material added.		

Analyzing Percent Relationships

At this point, you may be thinking, hey, this percent increase/decrease isn't so bad. But, I've got to warn you, it's like dating. Once you think you've got it down to a science, somebody comes along and knocks your socks off. With regards to the real world, percent changes don't always follow the same pattern. The information provided in Chart 1.4 doesn't follow the same pattern as in the previous examples.

Example #3: Let's look at an example that forces us to switch our procedure. Suppose you have 5 cubic yards of loose-stockpiled sand. What will the volume be when it is compacted?

From Chart 1.4, we see that the shrinkage factor for sand is 1.18. But wait! A multiplier of 1.18 indicates an 18% increase in volume. Since we're dealing with shrinkage we know that the volume should get smaller as the soil is compacted. Our multiplier should be less than 1, not bigger than 1. Just like a bad date, we've got mixed messages.

Since multiplying the factor won't work for us. We'll have to adjust the factor. For this example, we are going from a large volume of loose soil to a smaller volume of compact soil. So, we need our multiplier to be less than 1. Because the factor is 1.18, we need to convert this factor into a multiplier that's less than 1. The way to make the factor less than 1 is to divide it; the opposite of multiplying is dividing. Our factor will become $\frac{1}{1.18} \approx 0.847$. From our earlier work, we know that a multiplier of 0.847 is actually a 15.3% decrease $(1-0.847=0.153=15.3\%)$. We can use this multiplier to figure the volume of the compacted sand. The 5 cubic yards of loose-stockpiled sand will become $5 \times 0.847 \approx 4.24$ cubic yards.

Let's recap. We had the situation in which we were going from a large volume of loose soil to a small volume of compact soil. In the chart, we were given a factor that was larger than 1. We adjusted the factor so that it was less than 1 by dividing the factor. Then, we multiplied to get the volume of the compact soil.

Strategy

We have covered two situations with regard to calculating the volume of soil. In Chart 1.3, we started with the volume of compact soil and wanted to determine the volume of loose soil. The column of 1's in Chart 1.3 indicates that we are starting with the volume of the compact soil. In Chart 1.4, the column of 1's indicates that we are starting with the volume of the loose soil. The difference between the two situations is the direction we are going. In Examples #1 & 2, we started with compact soil (small volume) and ended with loose soil (big volume). In Example #3, we started with loose soil (big volume) and ended with compact soil (small volume). The following shows a mathematical way of looking at the situations using equations.

Let's start with the two basic equations: $L = kS$ and $S = kL$. Both equations are based on two main things, a larger amount and a smaller amount. To keep it simple, let's use L for the larger amount and S for the smaller amount. In our examples so far, L is like the volume of loose soil and S is like the volume of the compact soil. The loose soil always had a larger volume than the compact soil. The letter k stands for the multiplier or factor.

The difference between the two equations is the direction. If we are going from compact soil to loose soil, then we need to use the equation: $L = kS$. This equation says that we are starting with the smaller amount and ending up with the larger amount. The key thing for this formula is that the k-value needs to be larger than 1 because we are going from small to large. The multiplier has to be larger than 1 to go from small to large.

If we are going from loose soil to compact soil, then we need to use the equation: $S = kL$. This equation says that we are starting with a larger amount and ending with a smaller amount. The k-value needs to be less than 1 in order to go from large to small.

You've already encountered this type of relationship when you worked percents in the book. Recall $P = B \times R$. The letter P stands for part of the whole and B stands for the base. The base was usually larger than the part. And, the letter R stood for percent change, which was usually less than 1. Instead of using the letters P, B, and R, we're using L, S, and k because they have a simpler meaning. L stands for the larger amount and S stands for the smaller amount. And, the letter k is a letter that is quite often used for indicating multipliers.

Example #4: Suppose we have 7.8 cubic yards of loose clay-loam soil. What is the volume of the when the soil is compacted?

Since we are going from loose soil to compact soil (larger to smaller), we'll use the equation $S = kL$ and our multiplier needs to be less than 1. From Chart 1.4, the factor is given as 1.43; we need to convert the multiplier by dividing it: $k = \frac{1}{1.43} \approx 0.699$. Last, we need to multiply to get the volume of the compact soil: $S = kL \rightarrow S = 0.699 \times 7.8 \approx 5.5$ cubic yards.

Example #5: Okay, let's look back at Example 1 and use our new strategy. Recall, we're starting with excavating 2 cubic yards of sand. We then want to know the volume of the loose soil. From experience, we know that the smaller amount is the volume of the compact soil and the bigger amount is the volume of the loose soil. So, S is the volume of the hole and L is the volume of the pile of loose soil.

The next step is to determine which formula are we going to use, $L = kS$ or $S = kL$. Since we are starting with compact soil and ending with loose soil, we'll need to use $L = kS$. Because the k-value is with the S, this equation indicates that we're starting with the small volume and ending with the larger volume.

For sand, we see that the factor for swellage is 1.15, which is the size we want. Since we are going from small to big, we want the factor to be larger than 1. Now that we've got all of the numbers and the proper equation, let's plug them into our formula and calculate: $L = kS = 1.15 \times 2 = 2.3$ cubic yards. Our new strategy matches with the answer we got earlier.

Chart 1.5: Steps for Strategy		
1. Determine which is the bigger and smaller items.		
2. Determine the direction.	Small-to-Larger	Large-to-Smaller
3. Identify which formula to use.	$L = kS$	$S = kL$
4. Identify the factor given in the chart.	If factor is smaller than 1, convert it.	If factor is smaller than 1, use it.
	If factor is larger than 1, use it.	If factor is larger than 1, convert it.
5. Substitute numbers in and calculate.		

Example #6: Suppose we have 22 cubic yards of stock-piled loose clay. What will be the volume of the clay be when it's compacted?

We're starting with 22 cubic yards of loose clay. Chart 1.4 gives us a factor of 1.48. And, we want to know the volume when the clay is compacted. Since we are going from loose clay to compact clay, we'll use the equation, $S = kL$. Since the multiplier is larger than 1, we will need to convert it by dividing: $\frac{1}{1.48} \approx 0.676$. Let's plug in the numbers into the equation and calculate: $S = kL \rightarrow S = 0.676 \times 22 \approx 14.9$ cubic yards.

Shortcut

There is a shortcut to this process that is used out in the field. Whenever we have to convert the multiplier, we divided to make a new multiplier. Then, we multiplied. Shown in Example #6, we divided the factor to make the new multiplier, 0.676. Then, we multiplied to get the volume of the compact soil: 0.676×22. The shortcut that is used in the field is to just divide one time: $\frac{22}{1.48} \approx 14.9$. We get the same result.

Why does this shortcut work? Let's make a formula with all of the steps shown at one time. Starting with 0.676×22, let's plug in how we got the multiplier of 0.676. Recall, we divided the factor to get the 0.676: $\frac{1}{1.48} \approx 0.676$. So, instead of writing the 0.676, let's use the fraction. The formula, 0.676×22, will become $\frac{1}{1.48} \times 22$. You may recall from the textbook, that we can reorganize multiplication and division by combining the numbers into one fraction. So, the formula $\frac{1}{1.48} \times 22$ can be rewritten into one fraction as $\frac{22}{1.48}$. This fraction is telling us that we can divide the 22 cubic yards of loose clay by the factor that was given in Chart 1.4. As in step #4 of our strategy when the factor needs to be converted, we can just divide instead.

As in Example #4 where we have 7.8 cubic yards of loose clay-loam soil and a factor of 1.43, instead of converting the factor into a multiplier of 0.699, we can use division. The volume of the compact clay-loam soil is calculated by $\frac{7.8}{1.43} \approx 5.5$ cubic yards. This result matches the result by converting the factor. Dividing the factor saves us one calculation step but gives the same result.

Check Your Understanding

For the following questions, use the information in Chart 1.4.

C1. If you have 16 cubic yards of sandy-loam soil, what is the volume of the soil when it is compacted?

C2. If have 28 cubic yards of loose clay soil to develop a bank, what is the size of the bank when the clay soil is compacted? (this assumes that you don't spill or lose any soil in the process.)

C3. Okay, #2 wasn't so realistic. Suppose you estimate that you will lose 10% of the 28 cubic yards of clay in transport and construction of the bank. What is the size of the bank when the clay soil is compacted?

Answers: 1. 13 cubic yards. 2. 18.9 cubic yards. 3. 17.0 cubic yards.

Soil Excavation

Questions

1. A contractor gets a 4.5% discount from retail price at a supply company. If the retail price of his purchase totals $8520.50, what does she pay for the materials?
2. A bag of fertilizer is on sale for $26.39. If this is 20% less than the regular price, what is the regular price?
3. A contractor purchases 17,500 board feet of rough lumber. If there is a waste of 18% in milling, how many board feet of lumber are actually received?
4. A large box of nails costs $34.00. If there is a sales tax of 8.5%, what is the cost of a large box of nails?
5. Allowing for 12% waste, how many stepping stones must be ordered for a project that requires 1250 stepping stones?
6. A framer grabs a pizza for lunch at Mama Mary's pizzeria which advertises that their pizzas cost 10% less than the competition. If the pizzas down the street cost $9.50 each, what's the price of Mama Mary's pizzas?
7. A framer estimates that 1850 board feet of lumber are required for a project. Determine the board feet to be ordered if 10% is added for waste.
8. A contractor needs 634 bricks for a patio. Allowing for 8% waste, how many bricks should the contractor order?
9. A builder receives a 7.5% discount on plumbing and electrical supplies. If he purchases supplies that would retail for $5846.00, how much money does the discount save him?

Multi-step problems:

10. A worker purchases a power tool that regularly sells for $239.95. The tool is on sale for 15% off and there is a sales tax of 6%. How much does the worker pay for the tool?
11. A contractor's total bid of $47,995 includes the estimate, incidentals, and profit. The contractor adds 2% of the estimate for incidentals, and 8% of the estimate and incidentals for profit. What was the estimate?

12. If a flower bed will have a surface area of 64 square feet and a depth of 10 inches, how much soil is to be excavated?
13. If a walkway will have a depth of 2 inches and a surface area of 120 square feet, how much soil is be excavated?
14. For a foundation in clay-loam soil is to be made 4 feet deep with a surface area of 824 square feet. What is the volume of the loose soil that is to be hauled away? (Note: use Charts 1.3 and 1.6)

Chart 1.6 Computation of volume of excavated material (no swellage factor applied)	
Depth in inches and feet	Cubic yards per square surface foot
2 inches	0.006
6 inches	0.018
10 inches	0.031
1 foot	0.037
4 feet	0.148
6 feet	0.222
10 feet	0.369

Soil Excavation

CALCULATING THE COST OF BUILDING MATERIALS

The purpose of this section is to help you understand how framing lumber, trim stock and panel goods are bought and sold. We will explore three methods of costing lumber – board footage, linear footage and square footage. Before covering board footage, here are some of the terms and properties associated with framing lumber.

Framing Lumber - The Western Wood Products Association breaks framing lumber into three categories: dimensional lumber, structural decking and timber grades. We will primarily be working with dimensional lumber. For the purpose of this book, dimension lumber will be defined as ***2 by*** and ***4 by*** material or material ranging from 2-inches to 4 inches thick.

COMMON TERMS USED IN ASSOCIATION WITH FRAMING LUMBER:

Nominal Size – The dictionary defines nominal as "existing or being something in name only". In the context of framing lumber, nominal refers to framing lumbers width and thickness in name only, not actual size. We commonly refer to framing lumber as 2×4, 2×6, 2×8, 2×10, 2×12, etc.; however the **actual size or dressed size** of the lumber is of lesser dimension. Think of the nominal size as the size you pay for and dressed size the size you actually get. *See page 39 for a chart showing the nominal sizes and actual sizes of dimensional framing lumber.*

Standard Lengths – As noted in the table, standard lengths are offered in multiples of 2′, such as 8′, 10′, 12′, etc. There are also two specialty lengths used for wall studs, 88 $^{5}/_{8}$″ and 92 $^{5}/_{8}$″. When studs of these lengths are used with a single bottom plate and a double top plate, wall heights of 7′-8″ and 8′-0″ are attained.

Standard Sizes - Framing Lumber

Product	Description	Nominal Size		Dressed Dimensions		
				Thicknesses & Widths *(inches)*		
		Thickness *(inches)*	Width *(inches)*	Surfaced Dry	Surfaced Unseasoned	Length *(inches)*
Dimensional Lumber	S4S (Surfaced 4 Sides)	2"	2"	1 1/2"	1 9/16"	6' and longer generally shipped in multipals of 2'
		3"	3"	2 1/2"	2 9/16"	
		4"	4"	3 1/2"	3 9/16"	
			5"	4 1/2"	4 5/8"	
			6"	5 1/2"	5 5/8"	
			8"	7 1/4"	7 1/2"	
			10"	9 1/4"	9 1/2"	
			12"	11 1/4"	11 1/2"	
			over 12"	off 3/4"	off 1/2"	

FRAMING LUMBER IS BOUGHT AND SOLD THREE WAYS:

- **By the linear foot**
- **By the piece**
- **By the board foot**

LINEAR MEASURE

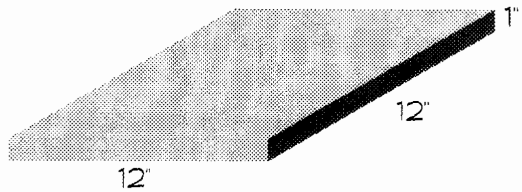
Linear is defined as a measure of length. This type of measurement does not take into consideration width and thickness, only length. The linear measurement of an 8′-0″ 2×12 is 8′-0″. Likewise, the linear measurement of an 8′-0″ 2×2 is also 8′-0″. At the retail level, framing lumber is typically sold by the piece or by the linear foot.

Example:

By the piece - 8′-0″ 2x4s cost $4.80 each
By the linear foot - 2x4s cost 60 cents per linear foot.

BOARD FOOT MEASURE

At the wholesale level, framing lumber is sold by the **board foot (BF)**. Unlike linear measurements, board footage is a measure of a board's volume. One board foot may be described as a piece of lumber 12 inches long, 12 inches wide and 1 inch thick. A board foot measurement is the ratio of a board's volume compared to the volume of 1 board foot.



One Board Foot

FIGURE 5-1

The volume of one board foot, measured in cubic inches, may be calculated as follows:

1 board foot =12"×12"×1"= 144 cu. in.

Therefore any piece of lumber equaling 144 cubic inches is one board foot. As previously stated, board footage is the ratio or relationship between the **volume (L×T×W)** of the board in question and the volume of one board foot. To better understand this concept, compare the volume of one linear foot of 2×4 to one-board foot.

Example 5-A: Calculate the number of board feet in one linear foot of 2×4.

Variables:

T = thickness expressed in inches
W = width expressed in inches
L = length expressed in inches

Equation:

$$\begin{array}{l} \textbf{2x4} \longrightarrow \\ \textbf{One board foot} \longrightarrow \end{array} \quad \frac{T''\times W''\times L''}{T''\times W''\times L''} = \frac{2''\times 4''\times 12''}{1''\times 12''\times 12''} = \frac{96 \text{ cu. in.}}{144 \text{ cu. in.}}$$

Solution: $\dfrac{96}{144} = \dfrac{8}{12} = .667$ BF

NOTE: ***Example 5-A is offered to help you gain a clear understanding of how board footage is calculated. In reality the equation may be greatly simplified.***

Simplifying the Board Foot Equation:

In order to simplify the equation, express length in feet rather than inches, otherwise board lengths normally given in feet must be converted to inches.

Now the board foot equation looks like this:

$$\textbf{Board Feet} = \frac{T'' \times W'' \times L'}{1'' \times 12'' \times 1'}$$

NOTE: ***One-foot and one-inch may be dropped, simplifying the denominator to 12 inches***

$$\textbf{Board Feet} = \frac{T'' \times W'' \times L'}{12''}$$

Example 5-C: Calculate the number of board feet in one linear foot of 2x4 using feet for length.

Variables:
L= length expressed in <u>feet</u> **(units)**
T = Thickness expressed in **inches**
W = width expressed in **inches**

Equation:

$$\frac{T'' \times W'' \times L'}{T'' \times W'' \times L'} \quad \text{or} \quad \frac{2'' \times 4'' \times 1'}{1'' \times 12'' \times 1'} \quad \text{Simplified} \quad \frac{2'' \times 4'' \times 1'}{12''} = .667 \text{ BF}$$

Try These - Calculate the board footage for one-foot lengths of the following dimension lumber sizes: Use the simplified equation

1. 2 x 4 = ____________BF
2. 2 x 6 = ____________BF
3. 2 x 8 = ____________BF
4. 2 x 10 = ____________BF
5. 2 x 12 = ____________BF

Try some more, this time with varying lengths:

6. One - 8′-0″ length of 2 x 8 = __________ BF
7. One - 12′-0″ length of 2 x10 = __________ BF
8. One - 14′-0″ length of 2 x 2 = __________ BF
9. One - 10′-0″ length of 4 x 10 = __________ BF
10. One - 8′-0″ length of 2 x 3 = __________ BF

INTRODUCING QUANTITY (# OF PIECES) INTO THE BOARD FOOT EQUATION

Usually, lumber orders will include more than one piece of material; therefore a quantity multiplier must be part of the calculation. To introduce the total number of pieces into the board foot equation, simply add the quantity (#) into the equation as below.

$$\boxed{\frac{\# \times T'' \times W'' \times L'}{12} = BF}$$

Example 5-C: Calculate the board footage of 8 pieces of 10′-0″, 2 x 4

$$\frac{8 \times 2 \times 4 \times 10}{12} = \mathbf{53.33\ BF}$$

Notice in the next set of questions, the lumber quantities and sizes are written differently. This notation is universally accepted, fast and reliable. The first number (12) indicates the number of pieces; the second (8) indicates the length of the pieces; and 4x12 indicates the size of the framing lumber.

12/8', 4x12 would indicate 12 pieces of 4x12, 8′ in length.

Try These:

11. 16/10', 2 x 4 = __________ BF
12. 10/12', 4 x 4 = __________ BF
13. 15/8', 6 x 8 = __________ BF
14. 13/8', 2x2 = __________ BF
15. 28/16', 2x14= __________ BF
16. 56/18', 2x3 = __________ BF

INTRODUCING COST TO THE BOARD FOOT EQUATION:

Lumberyards selling lumber to homebuilders, contractors, and remodelers typically quote lumber prices by the thousand, or by the cost of one thousand board feet. For example, 12 foot 2 x 4s might cost $575.00 per thousand board feet. You will see this written as 575M.

DEFINITION: ***M is the Roman numeral for one thousand, hence $575.00 per thousand board feet.***

In order to use this information in the equation we need to know the cost of one board foot. Simply move the decimal point three places to the left and you get the cost of one board foot, $00.575 or 57 ½ cents. Finally, multiply the calculated board footage by the cost per board foot to get the total cost.

$$\boxed{\left(\frac{\#pcs. \times L' \times T'' \times W''}{12}\right) \times \text{cost per BF} = \textbf{Board Foot Cost}}$$

Example 5-D:

What is the cost of 1/12 2×4 @575M?

Variables:
= Number of pieces

L = Length of Material

T = Thickness of Material

W = Width of Material

$ = Cost per Board Foot

Equation:

$$\frac{\# \times T \times W \times L}{12} = BF \times \$ = \textbf{Board Foot Cost}$$

Solution:

$$\frac{1 \times \cancel{12} \times 2 \times 4}{\cancel{12}} = 8 \times .575 = \$4.60$$

1- 12'-0" 2x4 equals 8 BF and costs $4.60 each

NOTE: ***As in the example, you may find it helpful to first calculate the board footage, write it down, and then multiply by the board foot cost, otherwise your equation will only reflect the cost.***

Try These:
Calculate the board footage and the price of the following lumber quantities:

17. 10/12' 2x2 @ 350M __________ BF $__________

18. 22/14' 2x8 @ 590M __________ BF $__________

19. 5/18' 4x16 @ 730M __________ BF $__________

20. 120/8' 2x4 @ 480M __________ BF $__________

21. 19/10' 2x12 @ 610M __________ BF $__________

22. 11/12' 4x4 @ 570M __________ BF $__________

23. 15/16' 4x6 @ 590M __________ BF $__________

24. 8/8' 2x6 @ 590M __________ BF $__________

25. 4/16' 4x10 @ 790M __________ BF $__________

26. 2/16' 2x3 @ 488M __________ BF $__________

PRICING BY THE LINEAR FOOT (LF)

Finish lumber and trim_– Finish lumber is typically used for trim work and is sold by the linear foot. Working in linear footage makes estimating quantity and price easy. Add up all of the lengths and number of pieces needed of a particular size and multiply times the cost per foot.

Example 5-E: You need 6/7′ 1x6 and 14/8′ 1x2 to complete a set of doorjambs. The 1x6 costs $1.20 per LF and the 1x2 costs .67¢ per LF. What is the total cost of the trim package?

6 × 7 × $1.20	= $50.40
14 × 8 × .67¢	= $75.04
Total	= $125.44

Trim boards, such as base trim, casing, crown molding, and other milled products, are most often priced by the linear foot.

Try These:

27. 12/16′ Colonial Base @ .78 cents/ft. __________

28. 15/8′ – 3 ½″ crown @ $9.38/piece __________

29. 22/8′, 11/7′, 14/12′ of 2 $^{1}/_{2}$″ colonial casing at .56 cents/L.F. __________

30. 15 pre-hung doors @ $56.00 each ____________

31. 9/14′ - 1×4 @ .68/ft, 12/8′ - 1x3 @ .59/ft, 23/16′- 1×6 @ $24.00 each.

PRICING BY THE SQUARE FOOT

Panel Products – Plywood, particleboard, drywall, wafer board, etc.; are sold by the square foot or by the sheet. Most sheet goods are manufactured in 4×8, 4×10 and 4×12 sizes and come in $^{1}/_{4}$″, $^{3}/_{8}$″, $^{1}/_{2}$″, $^{5}/_{8}$″, $^{3}/_{4}$″, and 1″ thickness.

CALCULATING THE COST OF PANEL PRODUCTS

By The Sheet - Multiply the number of sheets by the cost per sheet.

Example 5-F: 20 sheets of drywall costing $4.00 per sheet.

20 × $4.00 = $80.00

By The Square Foot – Start with the equation used to find the area of a rectangle.

Area = Lx W

Variables:
L = Length
W =Width
# = Number of pieces
$ = Cost per <u>square foot</u>

Equation:
L × W × # × $= Cost

Example 5-G: **Price 40 sheets of $^{1}/_{2}$″× 4 x 8 CDX plywood costing 500M**

NOTE: ***Prices per thousand in sheet goods do not take thickness into account. Therefore, move the decimal point three places to the left and multiply by the calculated square footage.***

4 x 8 x 40 x .50 = $640.00

Price per sheet 4 x 8 x .50 = $16.00

Try These:

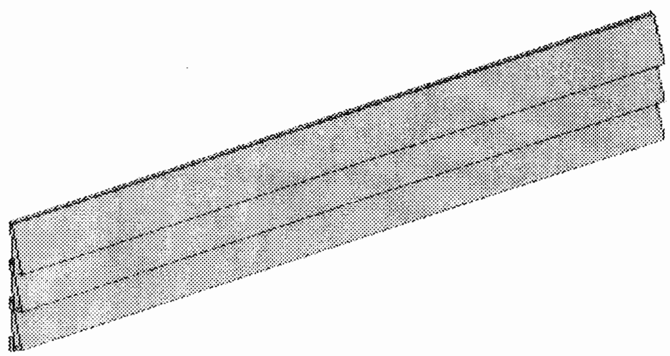
32. 20 sheets of $^1/_2$"x4'x12' drywall costing 200M ________________

33. 15 sheets of $^1/_4$"x4'x8' Masonite costing 320M ________________

34. 45 sheets of $^3/_4$"x4'x8' CDX plywood costing 560M ____________

LAP SIDING

Lap siding or clapboard siding is commonly used as an exterior covering on both residential and commercial structures. There are many types but the common thread is that each successive course of siding overlaps the course below.



Lapping successive courses of siding renders a waterproof shield, provided it is installed correctly. Some of the types of lap siding commonly used are cedar or redwood bevel siding, Hardy Plank concrete siding, and Smart Siding (formerly LP Siding).

Lap siding offers the contractor a real challenge when calculating quantity and price because, while covering a wall with a panel product would entail a fairly simple square footage calculation, lap siding is a plank, sold by the board foot, based on nominal width (different than the actual width), and to complicate things, the siding is overlapped.

Calculating waste takes experience and varies depending on the structure being sided, so we will focus on the fundamentals.

STEP-BY-STEP CALCULATION OF LAP SIDING

Step 1: Calculate the square footage of the wall or walls to be covered by multiplying the linear footage of wall times the wall height. Generally windows and doors are not subtracted from the calculation and serve as a waste factor. Let's say we calculated **2500 SF** of wall to be covered by our lap siding.

Step 2: Determine the siding width. Lap siding is sold in a variety of lengths, and widths such as 4″, 6″, 8″ and 10″. The widths are given as nominal sizes not actual sizes, so make sure you use the actual width for your calculations. For the sake of this exercise we will use 1×10″ bevel siding. Keep in mind that although 1×10 is the given or nominal size the actual size of the siding is **$^{11}/_{16}$" × 9 $^{1}/_{4}$"**. Siding is usually less than 1″ in thickness ranging from $^{1}/_{2}$″ to $^{3}/_{4}$″.

Step 3: Determine the lap. As the name implies lap siding is overlapped and you should follow the manufactures recommendations as to what the lap should be. For our example we will overlap each piece of siding 1″.

Step 4: Determine the number of linear feet of siding per square foot. One square foot = 12″ × 12″ One linear foot of siding equals 9 $^{1}/_{4}$″ × 12″, but the overlap of one inch reduces the actual coverage to 8 $^{1}/_{4}$″. So one linear foot of siding = 8 $^{1}/_{4}$″×12″.

In this example, one linear foot of siding is less than one square foot, so we need to know how many linear feet it will take to cover one square foot. Simply divide 12 by 8.25 (**12 + 8.25= 1.4545**). In other words, it takes 1.4545 LF of siding to cover 1SF of wall surface. You have created a **factor,** which can be used over and over again to calculate 10-inch bevel siding with a 1″ lap.

NOTE: ***Factors are covered in detail in Chapter 8.***

Step 5: Determine the total number of linear feet of siding. This is the easy part, simply multiply your siding factor (1.4545) times the total square footage, in this case 2500 SF. 1.4545× 2500 = 3636.25 LF

Step 6: Calculate the board foot cost of the siding. For this calculation we will use a nominal thickness of 1″, nominal width of 10″and a cost of $1400M. Set up the board foot formula and multiply times the cost per board foot as below. *Formula found on page 44*

$$\frac{1x10x3636.25}{12} = 3030.20BF \times \$1.40 = \mathbf{\$4242.28}$$

Example 5-H: You must calculate the cost of 6″nominal (5 $^{1}/_{2}$″ actual) beveled siding applied to 1340 SF of wall surface, use a $^{1}/_{2}$″ lap and a board foot cost of $1200M.

5.5 - .5 = 5

$$\frac{12}{5} = 2.4 \text{ (factor)}$$

$2.4 \times 1340 = 3216$ LF

$$\frac{6x3216}{12} = 1608BF$$

1608 x 1.2 = **$1929.60**

Try These:

35. Situation:

Total square footage to be covered = 4567 SF
Siding 8″ Bevel Siding, actual size 7 $^1/_2$″
Lap = 1″
Cost = 980M

Linear feet needed ________________

Board Footage________________

Cost________________

36. You must side a rectangular building with bevel siding. Here is what you know:

- The building measures 52′-0″×24′-0″.
- The wall height is 9′-0″
- There is a triangular gable at each end of the building with a base of 24′-0″ and an altitude of 6′-0″.
- The siding is 6″ cedar bevel actual size = 5 $^1/_2$″
- The lap is 1″
- The board foot cost is $1845M

Linear feet needed ________________

Board Footage________________

Cost________________

Chapter 5

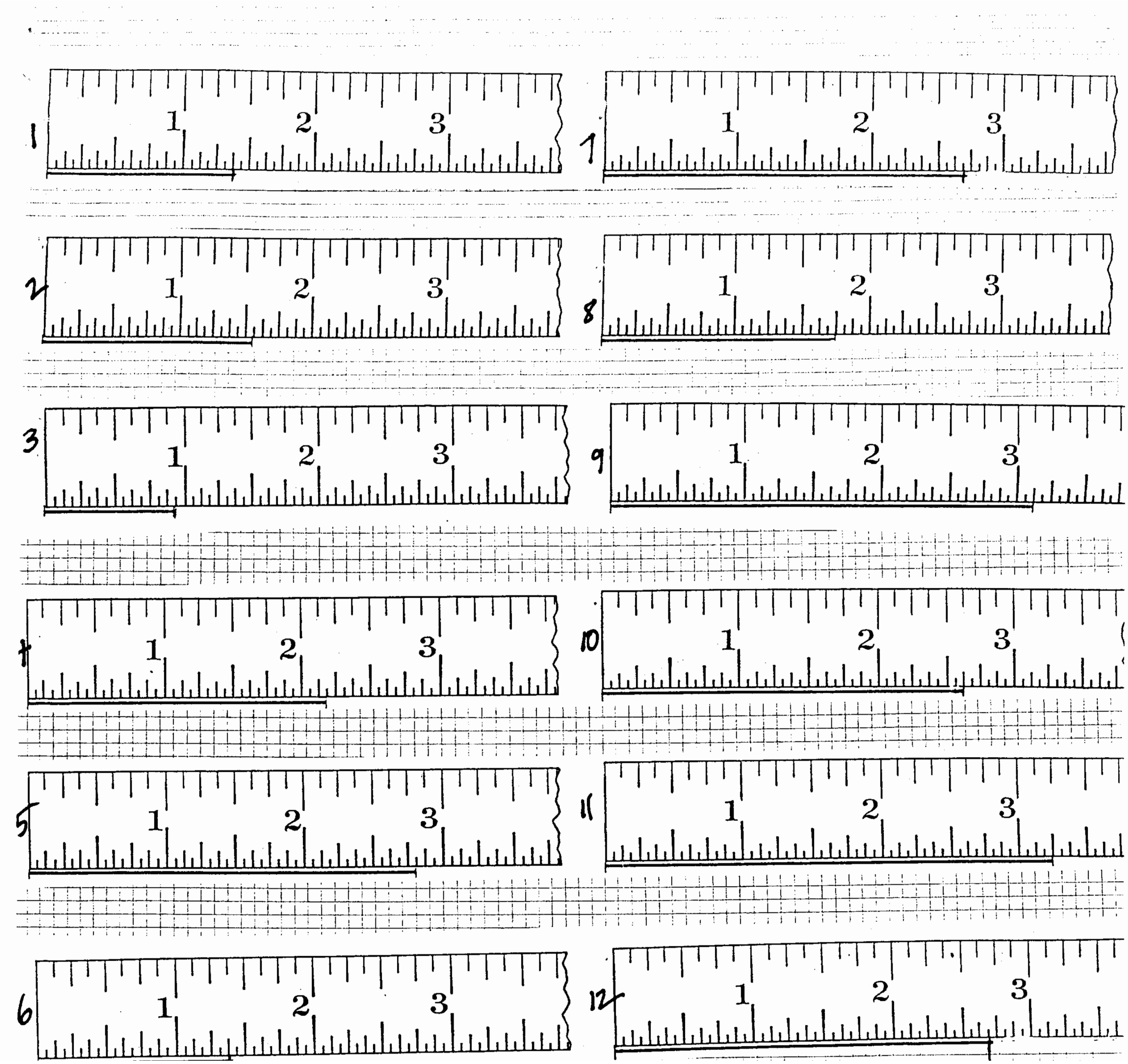
1. .6667
2. 1
3. 1.333
4. 1.667
5. 2
6. 10.6667
7. 20
8. 4.6667
9. 3.3333
10. 4
11. 106.67
12. 160
13. 480
14. 34.6667
15. 1045.3333
16. 504
17. 40 BF, $14.00
18. 410.67 BF,
    $242.29
19. 480 BF, $350.40
20. 640 BD, $307.20
21. 380 BF, $231.80
22. 176 BF, $100.32
23. 480 BF, $283.20
24. 64 BF, $37.76
25. 213.33 BF,
    $168.53
26. 16 BF, $7.81
27. $149.76
28. $140.70
29. $235.76
30. $840.00
31. $694.32
32. $192.00
33. $153.60
34. $806.40
35. 8431.38 LF
    5620.92 BF
    $5508.50
36. 4032 LF
    2016 BF
    $3719.52

Reading a Tape Measure

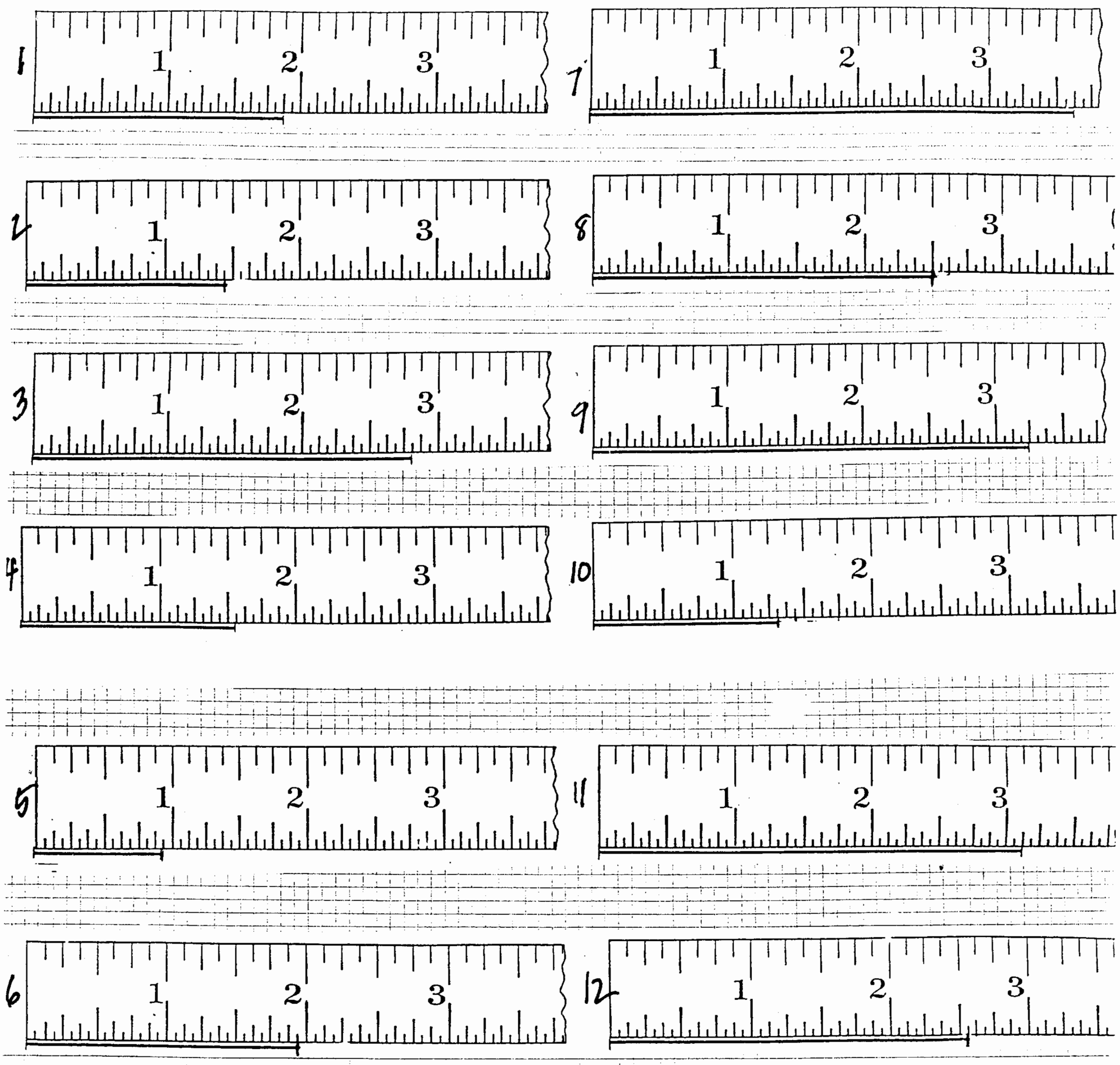
1. Students receive a hand-out at the beginning of class.
2. They are allowed one minute to complete as many readings as possible.
3. Each time the minute is up...they stop and record the number of correct answers and date it.

The exercise is designed to boost confidence in reading the tape measure while gaining in accuracy and speed.

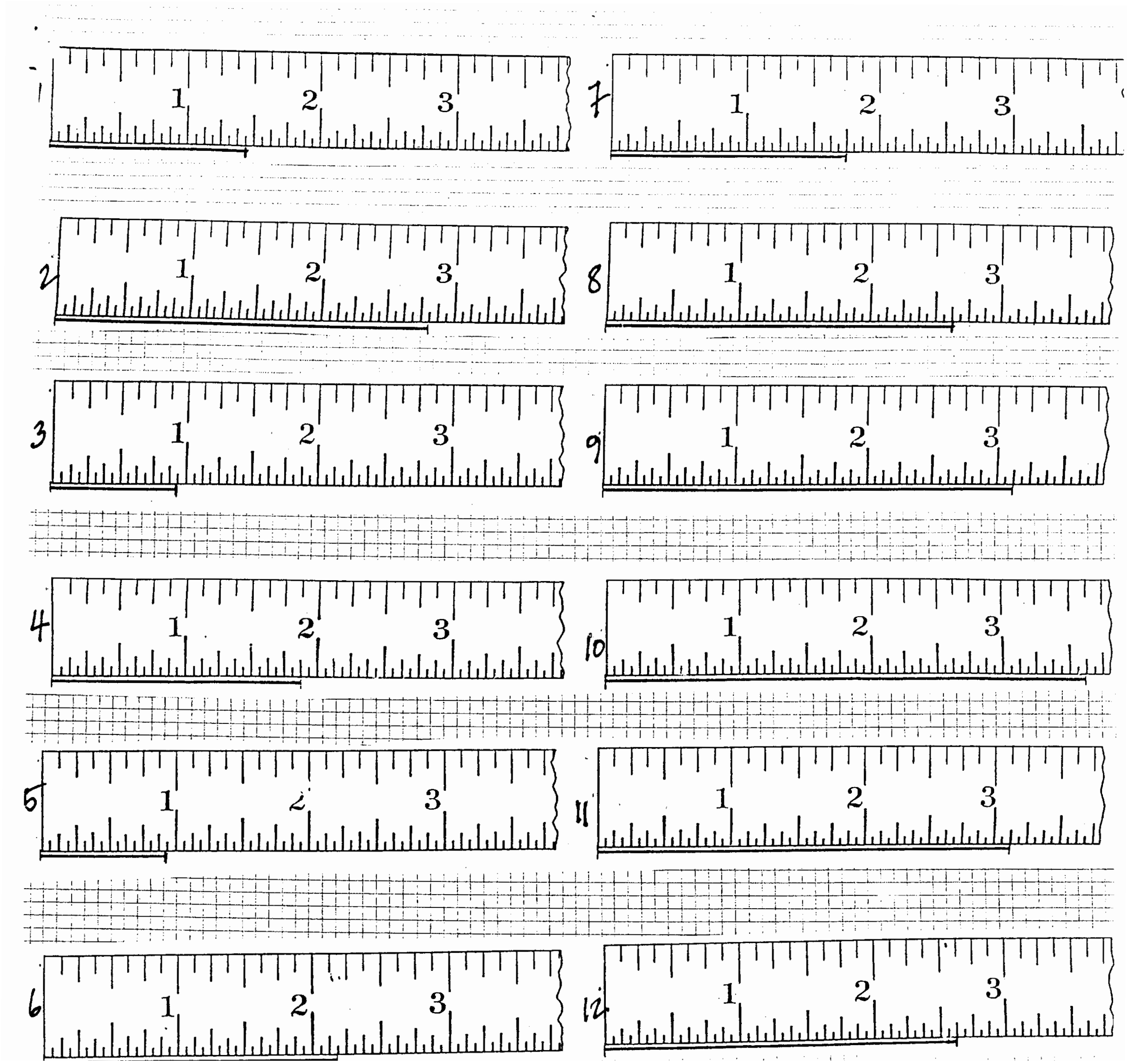
Reading a Tape Measure



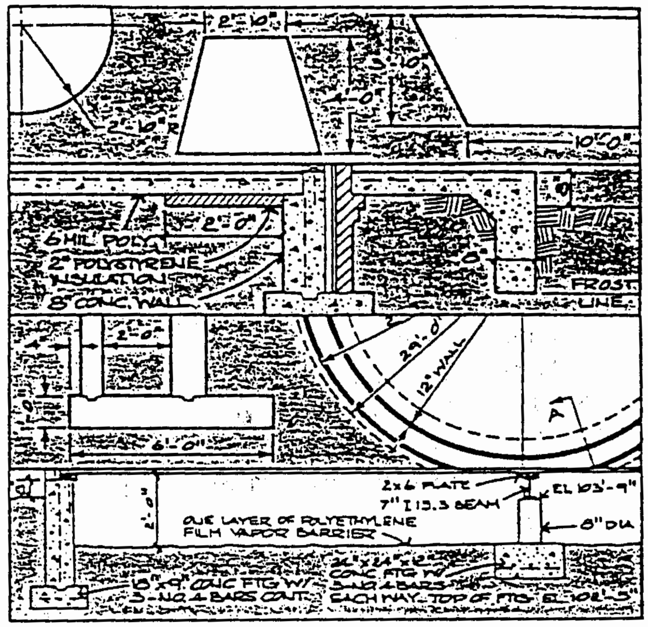
Reading a Tape Measure



Reading a Tape Measure



Concrete Formwork



A form builder encounters applications on the job that involve math and printreading skills. The applications range from calculating area and volume to determining placement of forms for foundation walls and footings.

Section 1, Math Fundamentals, includes information regarding conversions of decimal values to foot and inch equivalents and foot and inch values to decimal equivalents, area and volume calculations, and tread and riser calculations. Conversions and area and volume calculations are used frequently to determine the surface area of a wall form or volume of concrete required for a structure. Tread and riser calculations determine the number of treads and risers and their respective dimensions.

Section 2, Printreading Exercises, include questions related to five prints: Plot Plan, Slab-on-Grade Foundation, Crawl Space Foundation, Full Basement Foundation, and Heavy Construction Foundation. The prints contain information regarding foundation construction that is presented in chapters 3 to 5 of the text.

Section 3, Estimating Form Materials and Concrete, provides a step-by-step procedure for calculating the amount of form materials and volume of concrete required for a small residential structure. The Review Questions contain exercises for estimating the amount of form materials and volume of concrete for a full basement foundation.

SECTION 1

MATH FUNDAMENTALS

A form builder must have an understanding of basic math concepts to perform estimating and formwork operations. Mixed numbers and fractions, such as $4\frac{5}{8}''$ or $\frac{1}{2}''$, are routinely used in form construction. Decimal numbers, such as .55 or 101.8′, are used when referring to ratios or elevations. A form builder must convert decimal numbers to mixed numbers and fractions, and mixed numbers and fractions to decimal numbers to calculate area, volume, and tread and riser dimensions.

Decimal Foot to Inch Equivalents

Elevations on a plot or foundation plan are commonly expressed in decimal numbers, such as 45.2′ or 10.2′. A form builder must convert decimal numbers to the foot and inch equivalent for use with a standard tape or wood rule.

Mathematical conversion of decimal numbers to inches is accomplished by first determining the number of inches and then the fraction. The answers are combined to obtain the inch equivalent.

Example
Convert .83′ to an inch equivalent.

Solution

1. Multiply the decimal value by 12 to determine the number of inches and decimal part of an inch.

$$.83 \times 12 = \underbrace{9}_{\text{TOTAL NUMBER OF INCHES}}.\underbrace{96}_{\text{DECIMAL PART OF AN INCH}}$$

2. Multiply the decimal part of the inch by a commonly used denominator (for example 4, 8, or 16). A larger denominator value results in greater accuracy.

$$.96 \times \underbrace{16}_{\text{MULTIPLIER}} = 15.\underbrace{36}_{\text{DECIMAL REMAINDER}}$$

3. If the decimal remainder is .5 or greater, round the value preceding the decimal point up. If the decimal remainder is less than .5, round the value down. Use the rounded value as the numerator and the commonly used denominator in step 2 as the denominator. Reduce the fraction to lowest terms if possible.

15.36 rounded down to 15

$$\frac{\overbrace{15}^{\text{ROUNDED VALUE USED AS NUMERATOR}}}{\underbrace{16}_{\text{DENOMINATOR}}}$$

4. Combine the answers from steps 1 and 3 to obtain the inch equivalent.

$9'' + \frac{15}{16}'' = 9\frac{15}{16}''$

$.83' = 9\frac{15}{16}''$

When converting an elevation to an inch equivalent, the number preceding the decimal point is the total number of feet and the number following the decimal point is converted to inches.

Example
Convert 10.9′ to a foot and inch equivalent.

Solution

1. Multiply the value after the decimal point by 12 to determine the number of inches and decimal part of an inch.

$$.9 \times 12 = \underbrace{10}_{\text{TOTAL NUMBER OF INCHES}}.\underbrace{8}_{\text{DECIMAL PART OF AN INCH}}$$

2. Multiply the decimal part of an inch by a commonly used denominator (for example 4, 8, or 16). A larger denominator value results in greater accuracy.

$$.8 \times \underbrace{16}_{\text{MULTIPLIER}} = 12.\underbrace{8}_{\text{DECIMAL REMAINDER}}$$

3. If the decimal remainder is .5 or greater, round the value preceding the decimal point up. If the decimal remainder is less than .5, round the value down. Use the rounded value as the numerator and the commonly used demoninator in step 2 as the denominator. Reduce the fraction to lowest terms if possible.

12.8 rounded up to 13

$$\frac{\overbrace{13}^{\text{ROUNDED VALUE USED AS NUMERATOR}}}{\underbrace{16}_{\text{DENOMINATOR}}}$$

4. Combine the foot value (number preceding the decimal point in the example) with inch values.

$10' + 10'' + \frac{13}{16}'' = 10'\text{-}10\frac{13}{16}''$

$10.9' = 10'\text{-}10\frac{13}{16}''$

A conversion table is also used to convert decimal feet to inches. (See Appendix A for Conversion Table.) The decimal foot value to be converted is located in the table and the number of inches is read from the horizontal row above. The fractional part of an inch, in eighths, is read from the vertical column to the left. If a decimal foot value is not located in the conversion table, the mathematical conversion is used.

Example
Convert .22 ′ to an inch equivalent.

Solution

		Inches		
		0	1	2
8th of an Inch	0	.00	.08	.17
	1	.01	.09	.18
	2	.02	.10	.19
	3	.03	.11	.20
	4	.04	.13	.21
	5	.05	.14	.22
	6	.06	.15	.23
	7	.07	.16	.24

.22 ′ = $2\frac{5}{8}$ ″

Inch to Decimal Inch and Foot Equivalents

Fractions and mixed numbers are converted to decimal inch and foot equivalents for use in applications such as determining stair riser and tread dimensions. Decimal inch equivalents express the given value in terms of 1 ″, such as $\frac{3}{4}$″ = .75 ″. Decimal foot equivalents express the given value in terms of 12 ″, such as $\frac{3}{4}$″ = .06 ′.

A decimal inch equivalent is determined mathematically or by using a conversion table. (See Appendix A for Conversion Table.) To convert a fraction to a decimal inch equivalent, the numerator is divided by the denominator. A decimal point is placed before the equivalent.

Example
Convert $\frac{7}{8}$ ″ to a decimal inch equivalent.

Solution

7 ÷ 8 = .875

NUMERATOR
DENOMINATOR
DECIMAL INCH EQUIVALENT

$\frac{7}{8}$ ″ = .875 ″

A mixed number is converted to a decimal inch equivalent in a similar manner. The whole number remains the same and the fraction is converted to a decimal inch equivalent as in the previous example.

Example
Convert $8\frac{1}{4}$ ″ to a decimal inch equivalent.

Solution
$8\frac{1}{4}$ ″ = 8.25 ″

A decimal foot equivalent is determined by converting a fraction, whole number, or mixed number to a decimal inch equivalent. The decimal inch equivalent is divided by 12 to determine the decimal foot equivalent.

Example
Convert 10 ″ to a decimal foot equivalent.

Solution
10 ÷ 12 = .833
10 ″ = .833 ′

Example
Convert $8\frac{1}{2}$ ″ to a decimal foot equivalent.

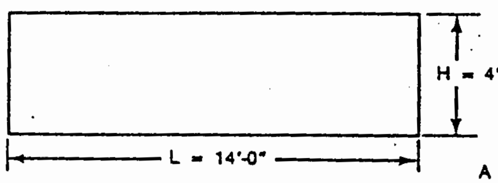
Solution
1. Convert $8\frac{1}{2}$ ″ to a decimal inch equivalent.
   $8\frac{1}{2}$ ″ = 8.5 ″
2. Convert 8.5 ″ to a decimal foot equivalent.
   8.5 ÷ 12 = .708
   $8\frac{1}{2}$ ″ = .708 ′

Area Calculation

Area is a measurement of space that a two-dimensional plane or surface occupies. Area calculations determine the amount of form sheathing material required or the area within building or property lines. Area is expressed in units such as square feet (sq ft) or square inches (sq in.). The area for various shapes is calculated by different formulas, but in general is determined by multiplying the length by the height or width. The area of a circle is determined by multiplying π (3.14) by the radius squared or .7854 times the diameter squared.

Squares, Rectangles, and Parallelograms. The area of a horizontal or vertical square or rectangular surface, such as a building site or form wall, is determined by multiplying the two outside dimensions. The area of a horizontal surface is determined by multiplying the width by the length, and a vertical surface is determined by multiplying the length by the height.

Example
Determine the area (A) of the rectangle.

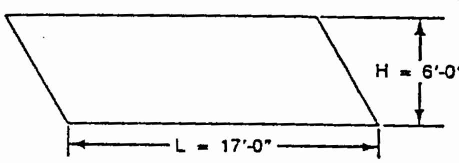


A = area
L = length
H = height

Solution

$A = L \times H$
$= 14'\text{-}0'' \times 4'\text{-}0''$
$Area = 56$ sq ft

Example
Determine the area (A) of the parallelogram.



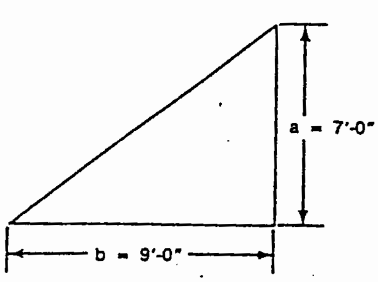
A = area
L = length
H = height

Solution

$A = L \times H$
$= 17'\text{-}0'' \times 6'\text{-}0''$
$Area = 102$ sq ft

Triangles. The area of a triangle is determined by multiplying the base dimension by the altitude and dividing by 2.

Example
Determine the area (A) of the triangle.



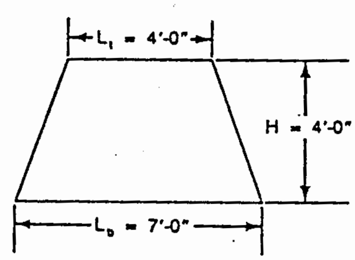
A = area
b = base
a = altitude

Solution

$$A = \frac{ba}{2}$$
$$= \frac{9'\text{-}0'' \times 7'\text{-}0''}{2}$$
$$= \frac{63}{2}$$
$Area = 31.5$ sq ft

Trapezoids. A trapezoid is a geometric shape having four sides in which two of the sides are parallel. Battered foundation walls or tapered pier footings are common designs incorporating a trapezoid shape. To determine the area of a trapezoid, add the length of the top and bottom and multiply the sum by the height. Divide the results by 2 to obtain the area.

Example
Determine the area (A) of the trapezoid.



A = area
H = height
L_t = length of top
L_b = length of bottom

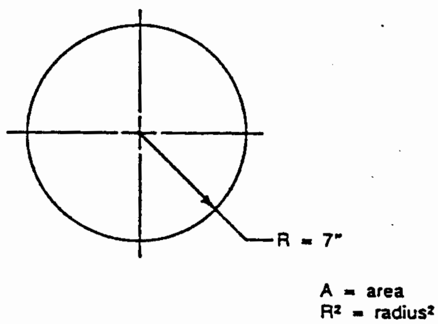
Solution

$$A = \frac{H\,(L_t + L_b)}{2}$$
$$= \frac{4'\text{-}0''\,(4'\text{-}0'' + 7'\text{-}0'')}{2}$$
$$= \frac{4'\text{-}0'' \times 11'\text{-}0''}{2}$$
$$= \frac{44}{2}$$
$Area = 22$ sq ft

Circles. The area of a circle is determined by using either the radius or the diameter of the circle. The radius is one-half the diameter and is measured from the center point to an edge of a circle. The diameter is the distance from one edge of a circle to the other and passing through the center point.

The area of a circle is calculated by multiplying π (3.14) by the radius squared (radius × radius). The area may also be determined by multiplying .7854 by the diameter squared (diameter × diameter).

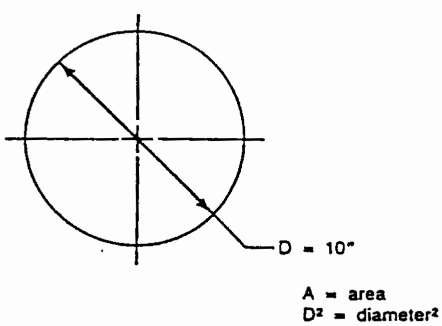
Example
Determine the area (A) of the circle using the radius of the circle.



Solution

$A = \pi R^2$
$= 3.14 \times (7'' \times 7'')$
$= 3.14 \times 49$
$Area = 153.86$ sq in.

Example
Determine the area (A) of the circle using the diameter of the circle.



Solution

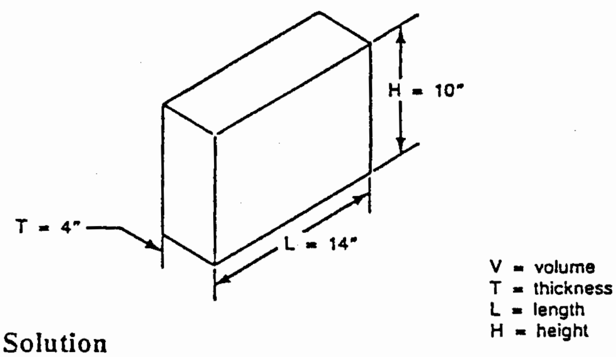
$A = .7854\ D^2$
$= .7854 \times (10'' \times 10'')$
$= .7854 \times 100$
$Area = 78.54$ sq in.

Volume Calculation

Volume is the amount of space that a three-dimensional figure or object occupies. Volume calculations are used to determine the amount of fill required for a building site or the amount of concrete required to form a footing or wall. Volume is expressed in cubic units, such as cubic feet (cu ft) or cubic yards (cu yd).

Rectangular Solids and Cubes. A rectangular solid is a six-sided solid object with a rectangular base. A cube is a solid object with six equal square faces. Examples of a rectangular solid or cube are a foundation wall, square pier footing, or square or rectangular column. The volume of a rectangular solid or cube is determined by multiplying its thickness, length, and height. The volume of a square column is determined by multiplying the width squared by the height.

Example
Determine the volume (V) of the rectangular solid.

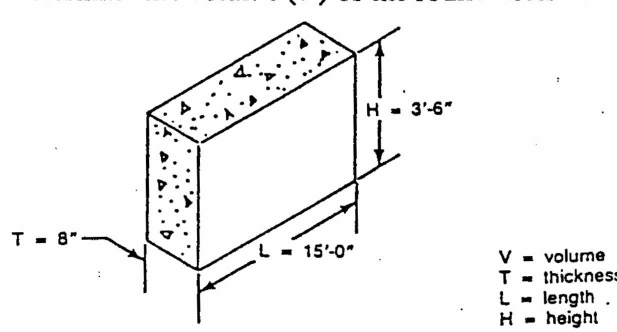


Solution

$V = T \times L \times H$
$= 4'' \times 14'' \times 10''$
$Volume = 560$ cu in.

When calculating large amounts of material, volume is commonly expressed in cubic feet or cubic yards. One cubic foot equals 1728 cubic inches and 1 cubic yard equals 27 cubic feet. When calculating volume for large amounts of material, thickness, length, and height are converted to decimal feet and then multiplied to obtain cubic feet. The result is divided by 27 to obtain cubic yards.

Example
Determine the volume (V) of the foundation wall.



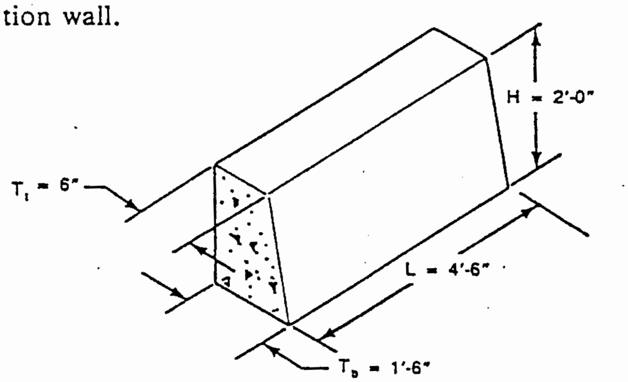
Solution

1. Convert the thickness (T), length (L), and height (H) to decimal foot equivalents.
$T = 8'' = .67'$
$L = 15'\text{-}0'' = 15.0'$
$H = 3'\text{-}6'' = 3.5'$
2. Calculate the volume of the foundation wall.
$V = T \times L \times H$
$= .67' \times 15.0' \times 3.5'$
Volume $= 35.18$ cu ft
3. Convert cubic feet to cubic yards.
Cu yd = *cu ft* $\div 27$
$= 35.18 \div 27$
Cubic yards $= 1.3$

Frustums of Pyramids. A frustum is a pyramid cut off parallel to its base. The sides of a frustum, such as a tapered pier footing, are trapezoids. The volume of a frustum with one battered side, such as a battered foundation wall, is determined by multiplying the average thickness by the height and length. The approximate volume of a frustum with four battered sides is determined by adding the areas of the top and bottom and dividing by 2 and then multiplying by the height.

Example

Determine the volume (V) of the battered foundation wall.

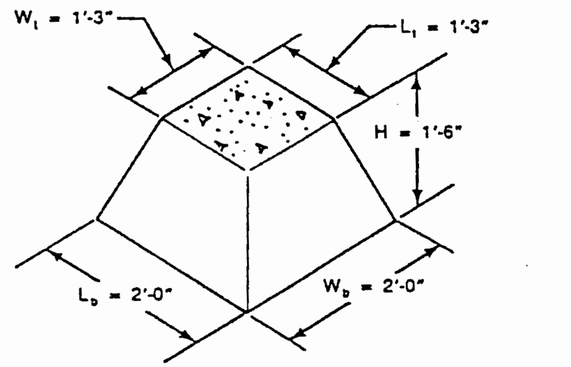


Solution

1. Convert the top thickness (T_t), bottom thickness (T_b), height (H), and length (L) to decimal foot equivalents.
$T_t = 6'' = .5'$
$T_b = 1'\text{-}6'' = 1.5'$
$H = 2'\text{-}0'' = 2.0'$
$L = 4'\text{-}6'' = 4.5'$
2. Calculate the volume of the battered foundation wall.
$$V = \frac{T_t + T_b}{2} \times H \times L$$
$$= \frac{.5' + 1.5'}{2} \times 2.0' \times 4.5'$$
$$= 1.0' \times 2.0' \times 4.5'$$
Volume $= 9$ cu ft
3. Convert cubic feet to cubic yards.
Cu yd = *cu ft* $\div 27$
$= 9 \div 27$
Cubic yards $= .33$

Example

Determine the volume (V) of the tapered pier footing.



Solution

1. Convert dimensions to decimal foot equivalents.
$W_t = 1'\text{-}3'' = 1.25'$
$L_t = 1'\text{-}3'' = 1.25'$
$W_b = 2'\text{-}0'' = 2.0'$
$L_b = 2'\text{-}0'' = 2.0'$
$H = 1'\text{-}6'' = 1.5'$
2. Determine the areas of the top (A_t) and bottom (A_b).
$A_t = L_t \times W_t$
$= 1.25' \times 1.25'$
Area of top $= 1.56$ sq ft
$A_b = L_b \times W_b$
$= 2.0' \times 2.0'$
Area of bottom $= 4.0$ sq ft

3. Determine the volume of the tapered pier footing.

$$V = \frac{A_t + A_b}{2} \times H$$

$$= \frac{1.56 + 4.0}{2} \times 1.5'$$

$$= 2.78 \times 1.5'$$

$$Volume = 4.17 \text{ cu ft}$$

4. Convert cubic feet to cubic yards.

$$Cu\ yd = cu\ ft \div 27$$

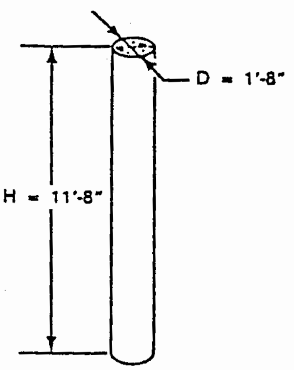
$$= 4.17 \div 27$$

$$Cubic\ yards = .15$$

Cylinders. A cylinder is a solid object with a circular cross-sectional area. Round columns and piers are cylinders. The volume of a cylinder is determined by multiplying the cross-sectional area by the height. The cross-sectional area is determined by using the diameter or radius.

Example

Determine the volume (V) of the round column.



V = volume
D² = diameter²
H = height

Solution

1. Convert the diameter (D) and height (H) to decimal foot equivalents.

$D = 1'\text{-}8'' = 1.67'$

$H = 11'\text{-}8'' = 11.67'$

2. Determine the volume of the round column.

$$V = .7854 \times D^2 \times H$$

$$= (.7854 \times 1.67^2) \times 11.67'$$

$$= (.7854 \times 2.79) \times 11.67'$$

$$= 2.19 \times 11.67'$$

$$Volume = 25.56 \text{ cu ft}$$

3. Convert cubic feet to cubic yards.

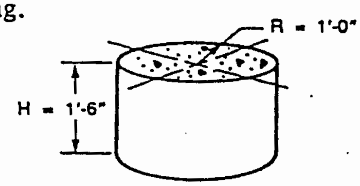
$$Cu\ yd = cu\ ft \div 27$$

$$= 25.56 \div 27$$

$$Cubic\ yards = .95$$

Example

Determine the volume (V) of the circular pier footing.



V = volume
R² = radius²
H = height

Solution

1. Convert the radius (R) and height (H) to decimal foot equivalents.

$R = 1'\text{-}0'' = 1.0'$

$H = 1'\text{-}6'' = 1.5'$

2. Determine the volume of the circular pier footing.

$$V \doteq \pi R^2 \times H$$

$$= (3.14 \times 1.0^2) \times 1.5'$$

$$= (3.14 \times 1.0) \times 1.5'$$

$$= 3.14 \times 1.5'$$

$$Volume = 4.71 \text{ cu ft}$$

3. Convert cubic feet to cubic yards.

$$Cu\ yd = cu\ ft \div 27$$

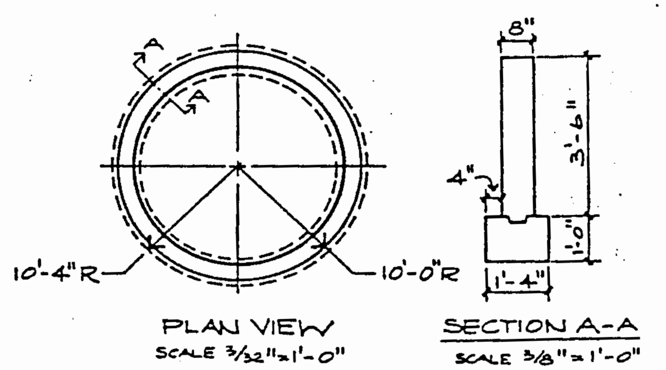
$$= 4.71 \div 27$$

$$Cubic\ yards = .17$$

Curved Walls and Footings. Curved walls and footings are used in the construction of storage tanks and silos. When calculating the volume of a curved wall or footing, the circumference (distance around the outside edge) is multiplied by the thickness and height.

Example

Determine the volume (V) of the curved wall and footing. Include the keyway in the footing calculations.



V = volume
R = radius
C = circumference
W = width
H = height

Solution

1. Convert the curved wall dimensions to decimal foot equivalents.
   $R = 10'\text{-}0'' = 10.0'$
   $H = 3'\text{-}6'' = 3.5'$
   $T = 8'' = .67'$
2. Determine the circumference (C) of the curved wall.
   $C = 2\,\pi\,R$
   $= 2 \times 3.14 \times 10.0'$
   Circumference $= 62.8'$
3. Determine the volume of the curved wall.
   $V = C \times T \times H$
   $= 62.8' \times .67' \times 3.5'$
   Volume = 147.27 cu ft
4. Convert cubic feet to cubic yards.
   Cu yd = *cu ft* ÷ 27
   = 147.27 ÷ 27
   Cubic yards = 5.45
5. Convert the footing dimensions to decimal foot equivalents.
   $R = 10'\text{-}4'' = 10.33'$
   $H = 1'\text{-}0'' = 1.0'$
   $W = 1'\text{-}4'' = 1.33'$
6. Determine the circumference of the footing.
   $C = 2\,\pi\,R$
   $= 2 \times 3.14 \times 10.33'$
   Circumference $= 64.87'$
7. Determine the volume of the footing.
   $V = C \times T \times H$
   $= 64.87' \times 1.33' \times 1.0'$
   Volume = 86.28 cu ft
8. Convert cubic feet to cubic yards.
   Cu yd = *cu ft* ÷ 27
   = 86.28 ÷ 27
   Cubic yards = 3.2

Tread and Riser Dimensions

Tread and riser dimensions are determined before the stairway forms are constructed. The tread is the horizontal surface of a step. The riser is the vertical member between two steps. Riser height is determined by dividing the total rise of a stairway by the number of risers. Tread depth is determined by dividing the total run of the stairway by the number of treads. The number of treads in a stairway is one less than the number of risers.

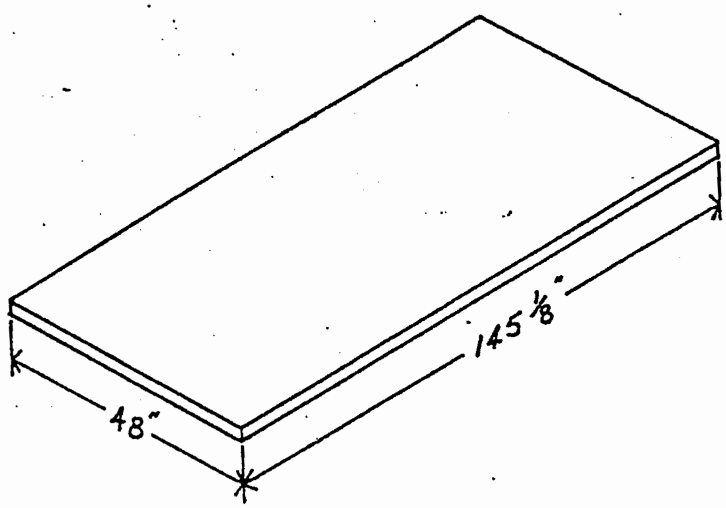
Example

Determine the number of treads and risers and the riser height and tread depth of a stairway. The total rise is 6′-7″ and the total run is 8′-6″. The riser height should be between 7″ and 7½″ and the tread depth should be 10″ minimum.

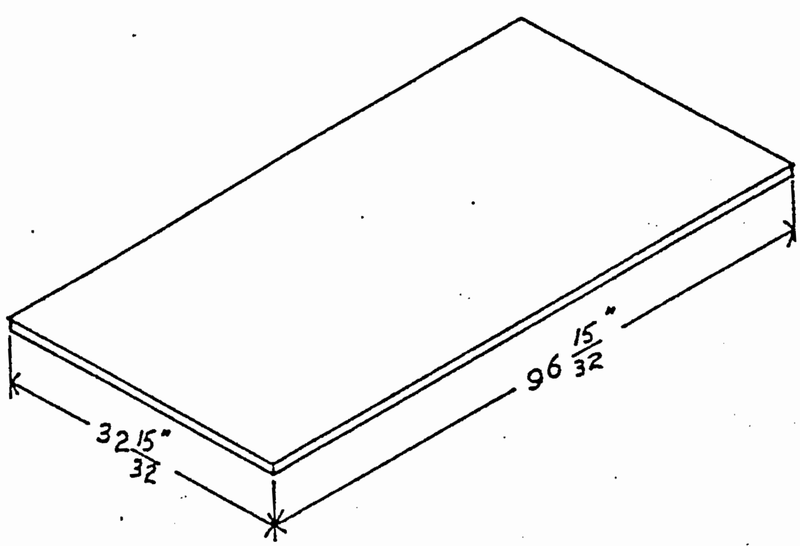
Solution

1. Convert the total rise to inches.
   Total rise =
   (*no. of feet* × 12) + *no. of inches*
   = (6′ × 12) + 7″
   Total rise = 79″
2. Determine the number of risers by dividing the total rise by the minimum desired riser height. Disregard the decimal remainder.
   No. of risers =
   total rise ÷ *minimum riser height*
   = 79 ÷ 7
   = 11.28
   No. of risers = 11
3. Determine the exact riser height by dividing the total rise by the number of risers.
   Riser height = *total rise* ÷ *no. of risers*
   = 79 ÷ 11
   Riser height = 7.18″
4. Convert the decimal inch value to a fractional equivalent.
   $7.18'' = 7\frac{3}{16}''$
5. Convert the total run to inches.
   Total run =
   (*no. of feet* × 12) + *no. of inches*
   = (8′ × 12) + 6″
   Total run = 102″
6. Determine the tread depth by dividing the total run by the number of treads.
   Tread depth = *total run* ÷ *no. of treads*
   = 102″ ÷ 10
   Tread depth = 10.2″
7. Convert the decimal inch value to a fractional equivalent.
   $10.2'' = 10\frac{3}{16}''$

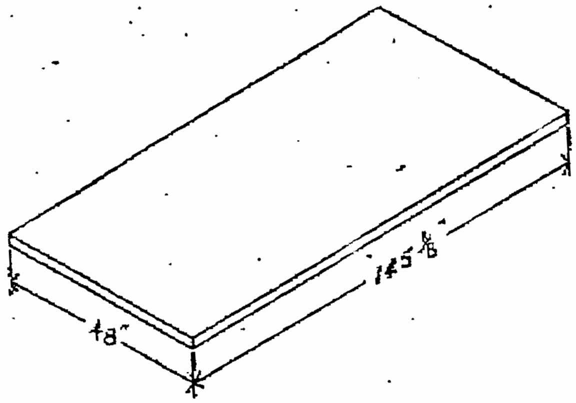
13. How many strips 48 in. long and 13 7/16 in. wide can be sheared from this sheet of 15 gage steel?



14. How many 7 3/4 in. square pieces can be cut from this sheet?



13. How many strips 48 in. long and 13 7/16 in. wide can be sheared from this sheet of 15 gage steel?



$$\frac{145.125}{13.4375}$$

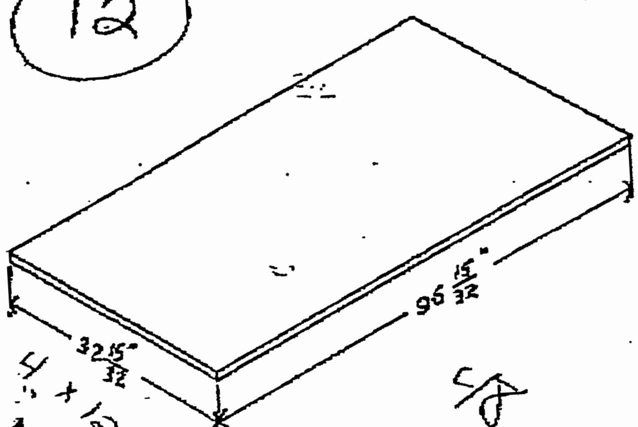
$10\frac{13}{16}$

10

14. How many 7 3/4 in. square pieces can be cut from this sheet?

$$\frac{96.46875}{7.75}$$

12



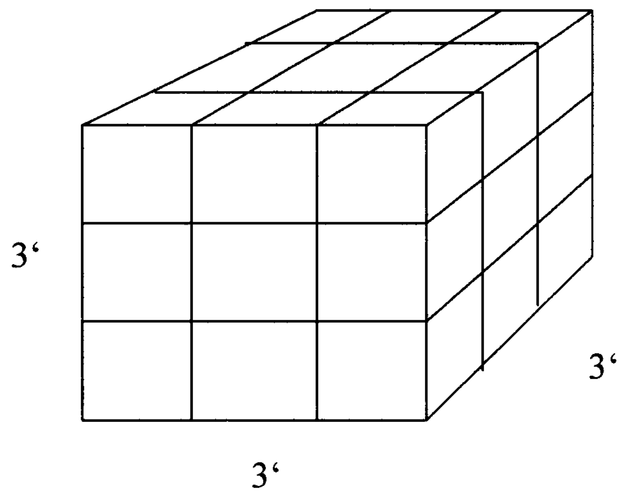
4 x 12

46875

5/8

Concrete Volume

The volume of a yard of concrete is the amount in a slab measuring 3 ft. x 3 ft. x 3 ft.



Slabs are a multiple or fraction of this three foot depth. For instance, if you set this 3' x 3' x 3' block on the ground, it covers 9 sq. ft., so in a three foot slab, one yard covers 9 sq. ft. If you had an 81 sq. ft. slab, you would divide 9 into 81, and order 9 cubic yards of concrete.

What if you had a slab one foot deep? If you took the block above, and laid out all the one foot cubes, you would discover that there are 27 cubic feet to a cubic yard. Therefore, if the slab is one foot deep, one yard of concrete will cover 27 square feet.

If the slab is 6 inches deep, cut the 27 blocks in two (6 inc. is one half of a foot), and you have 54 blocks covering 54 square feet.

If the slab is 4 inches thick, you would cut the 27 one foot cubes into 3 equal sections (4" x 3 = 12", or 1 foot), and you would have 81 one foot square blocks. One yard of concrete covers 81 sq. ft. at a 4" depth.

At 3 inches deep, you would cut them into 4 equal sections (3" x 4 = 12") This would give you 108 one foot square blocks, three inches deep.

If you do still not understand this principle, inform your instructor you need special tutoring.

Concrete Volume

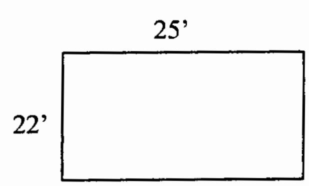
Below is a table, for your convenience. Using the above formula you will see how these numbers were derived.

	a depth of:	**with this square feet of coverage:**
One yard of concrete will cover:	1 in.	324
	2 in.	162
	3 in.	108
	4 in.	81
	5 in.	65
	6 in.	54
	7 in.	46
	8 in.	40
	9 in.	36
	10 in.	32.5
	11 in.	29.5
	12 in.	27
	18 in.	18
	24 in.	13.5

FIGURING YOUR JOB

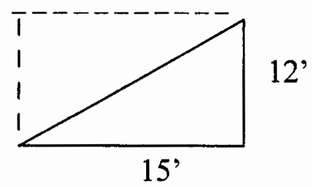
Now that you understand depth and coverage, you need to be able to determine the footage and depth of your slab.

Your job today is a carport measuring 22 x 25 feet. It will be 4 in. deep. To determine the area of a square or rectangle, multiply the length by the width – or 22' x 25' = 550 sq. ft.



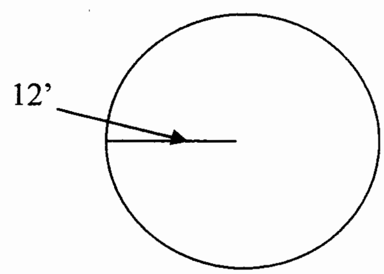
We know at 4 in. deep, a yard of concrete will cover 81 sq. ft. So how many yards do you need? Divide 81 into 550, and you get 6.79 yards of concrete. You decide to order 7.25. Good move – you are brilliant! There could be variances in the subgrade, and ordering back a small clean-up load can be very costly. More importantly, you will have a better quality job with no cold joints, and you saved valuable time on the end product.

If it's a triangle shape you are pouring, measure the sides (not the hypotenuse – the long side), multiply them and divide by two (because it is one half the size of a square or rectangle).



The sides are 12 and 15 feet. 12' x 15' = 180 sq. ft., divided by 2 = 90 sq. ft. At 4 in. deep, 90 sq. ft. divided by 81 sq. ft. = 1.1 yards. Order extra.

If it is a circle, the area is πr^2 (remember?), where $\pi = 3.14$ and r = the radius, or half the diameter.



The radius is 12 ft. – 3.14 x 144 sq. ft. (12^2 = 12' x 12' = 144 sq. ft.) = 452.16 sq. ft. At 6" deep, 452.16 sq. ft. divided by 54 sq. ft. of coverage = 8.4 yards.

Concrete Volume

1a. Convert .672 inches to the nearest 1/16-inch.

1b. Convert 7.821 inches to the nearest 1/32-inch.

1c. Convert 1.437 inches to the nearest ¼ inch.

2. Find the fraction ½ way between 5/32 and 3/16.

3a. Change 18.342 feet to feet and inches, to the nearest 1/16 inch.

3b. Change 8 ft. 3 inches to decimal feet.

4a.

	6 ft	2	3/4	in
	9 ft	7	1/8	in
	5 ft	3	5/16	in
+	4 ft	7	9/32	in

4b.

	10 ft			
-	3 ft	4	3/8	in

4c.

	23 ft	2	1 / 4	in.
-	5 ft	6	11 / 32	in.

5a. 7 ft 9 in x 1 / 2 =

5b. 6 1 / 4 ft divided by 5 / 8 in =

6a. Write in words this angle measure: 8° 29' 13"

6b. A right angle measures how many degrees?

6c. A straight angle measures how many degrees?

6d. What is the measure of the angle complementary to a 32° angle?

7a.

	42°	38'	51"
+	21°	47'	23"

7b.

	90°		
-	25°	16'	4"

Carpentry

8a. If 3 pounds of common bright 8d nails cost $ 1.19, what is the price of 7 pounds of these nails?

8b. A construction crew of six carpenters (all working at the same rate) can build a house in 5 weeks.
How long would it take a crew of 10 carpenters to build the same house?

8c. What is the slope of a shed roof having a run of 8' 0" and a rise of 2' 0"?

9a. 8 square feet equals how many square inches?

9b. 54 square yards equals how many square feet?

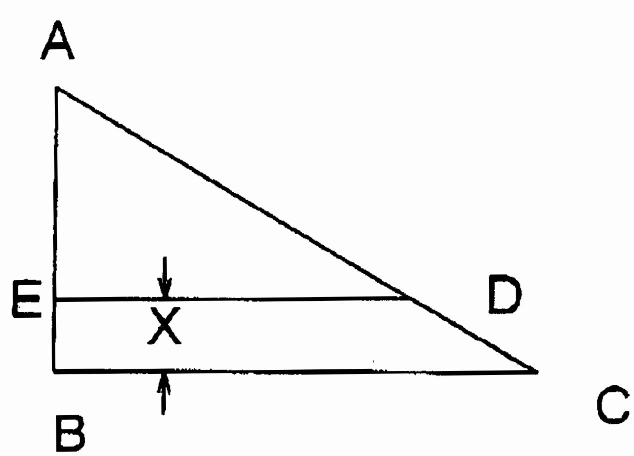
9c. 2880 cubic inches equals how many cubic feet?

9d. 24 cubic yards equals how many cubic feet?

10. It is estimated that 2,500 board feet of roof boards are needed. If an additional 20% is allowed for waste, find the total required.

11. Triangle ABC is similar to triangle AED. Find the distance X.

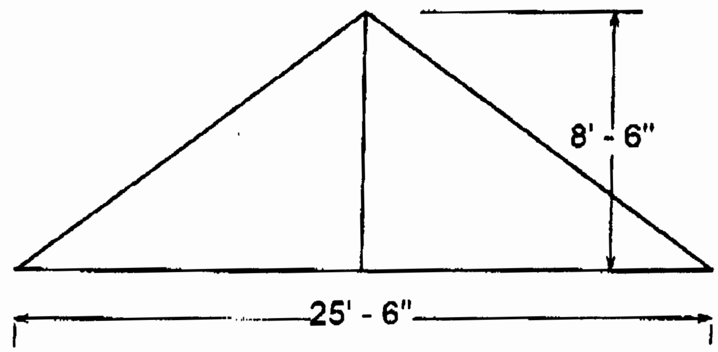
AB - 5"
ED - 6.2"
BC - 8.4"



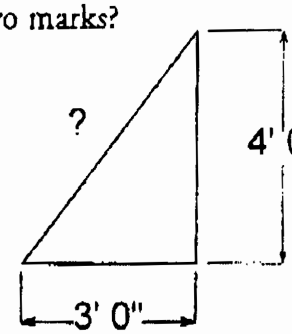
12a. Determine the common rafter length in a roof having a 5/12 slope and a rise of 16 feet. (Give answer in feet and inches to the nearest 1/16 inch.)

12b. A rectangular foundation has sides that measure 32'- 0" long and 26'-0" wide. To check that the foundation is a true rectangle, what would the diagonal measure? (Give answer in feet and inches to the nearest 1/16 inch.)

12c. A roof has a rise 8' 6" and a run of 12' 9". To the nearest 1/8 inch find the length of the roof rafters as shown.

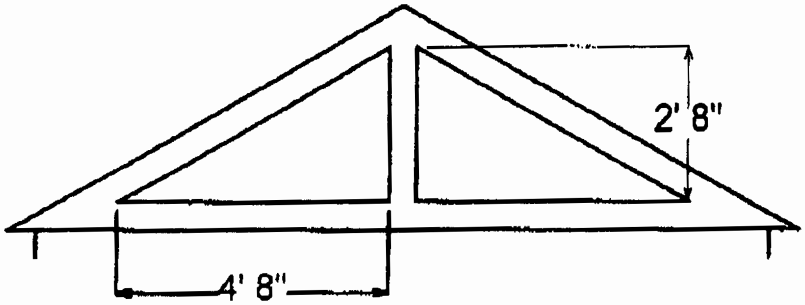


13. To test if a 90° angle exists at a corner where two walls meet, a carpenter measured 3'-0" along the base of one wall and 4'-0" along the base of the other wall. What should be the distance between these two marks?

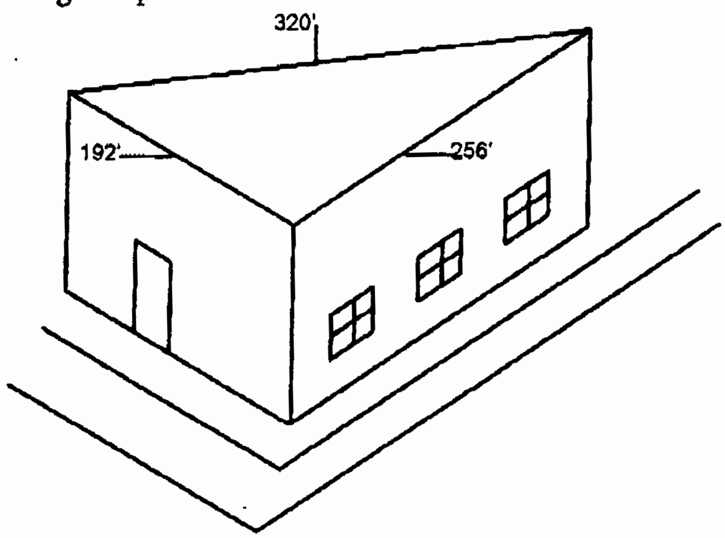


14a. Find the perimeter of each window below.

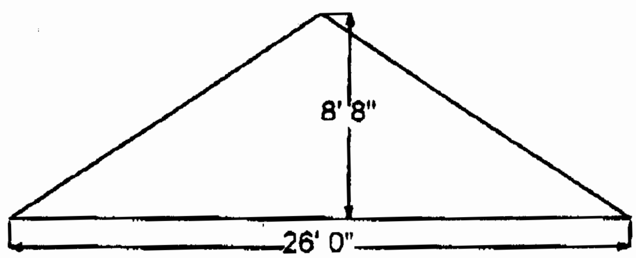
14b. Find the area of the glass in <u>each</u> of the windows.



15. A triangular building on a city lot has the dimensions shown. What is the area of each floor of the building in square feet?

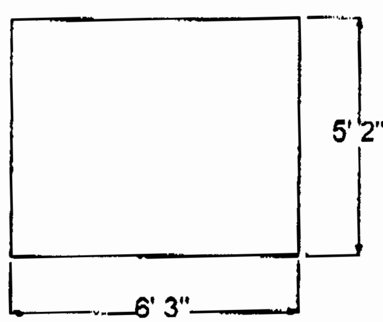


16. Using a framing square , find the length of a common rafter (without allowing for tail or overhang) in the building shown.



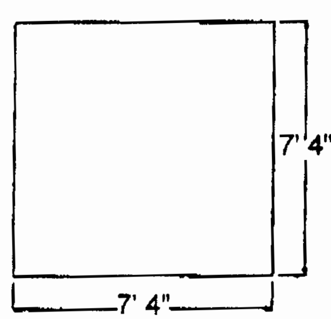
17a

1. perimeter =
2. area in sq. ft =
3. area in sq. in. =



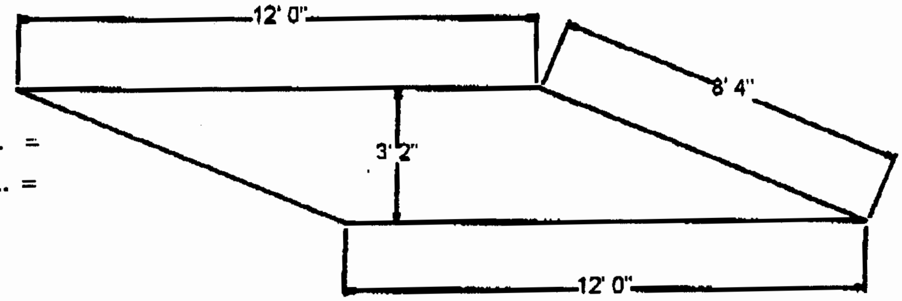
17b

1. perimeter =
2. area in sq. ft =
3. area in sq. in. =



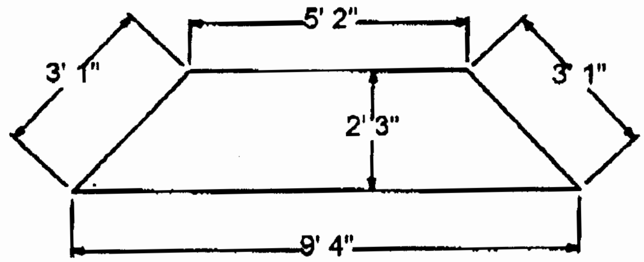
17c.

1. perimeter =
2. area in sq. ft. =
3. area in sq. in. =

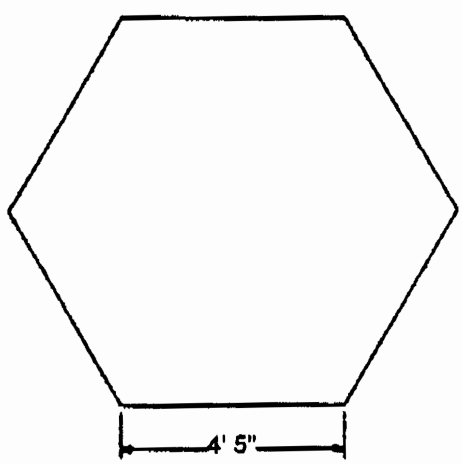


17d.

1. perimeter =
2. area in sq. ft. =
3. area in sq. in. =

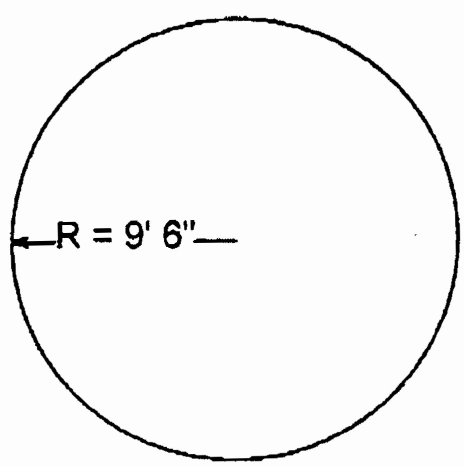


18. Find the area of a regular hexagon with each side 4' 5".

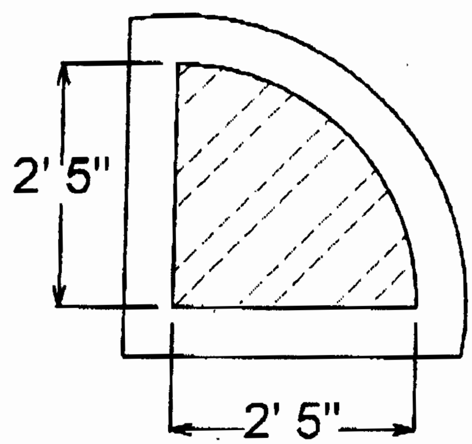


19a.

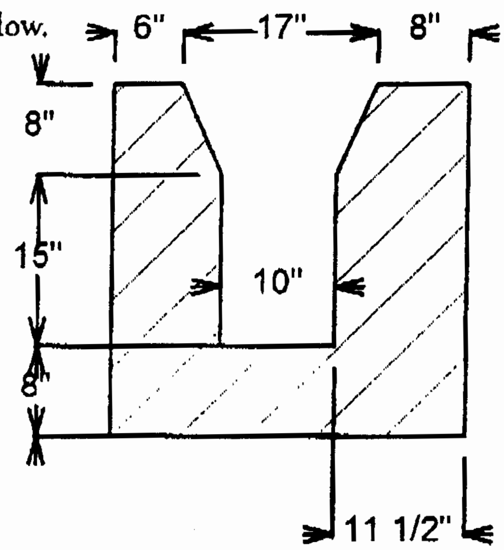
1. area =
2. circumference =
USE 3.14 for Pi.



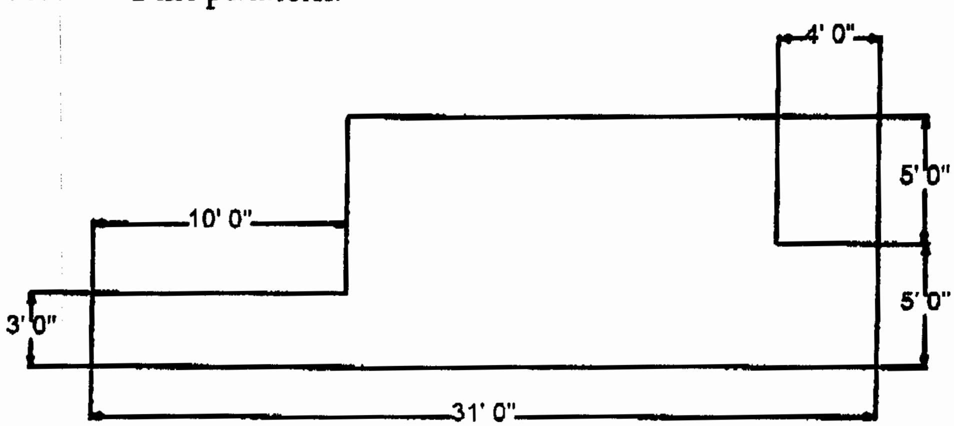
19b. 1. How many square inches of glass are in the window below?
2. To the nearest 1/8 inch, what is the perimeter of the window?



20a. Find the area of the section view below.

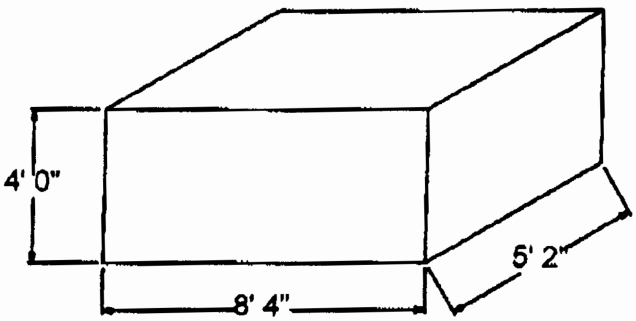


20b. Find the perimeter.



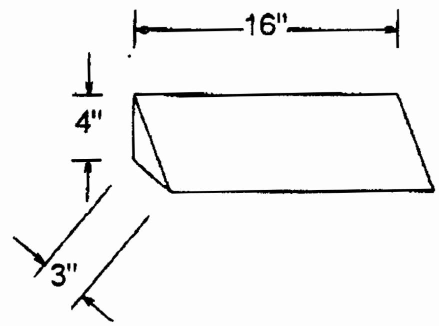
21a. Lateral surface area = (1)______________sq. ft. (2)______________sq. in.

volume = (3)______________cu. Ft. (4)______________cu. In.

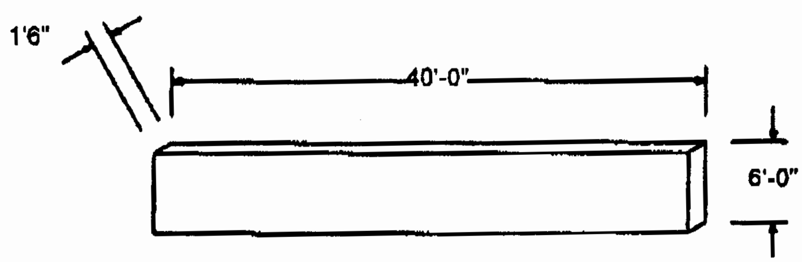


21b. (1) Total surface area =

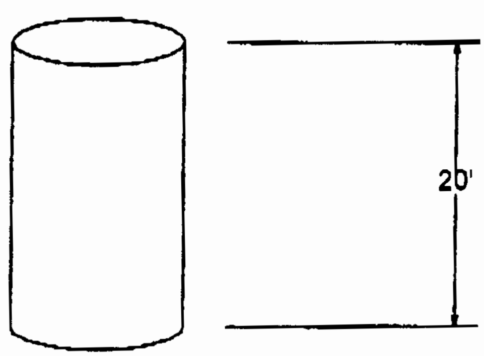
(2) Volume =



21c. How many yards of concrete are needed for a rectangular wall 40'-0" long, 6'-0" high, and 1 ½" thick?



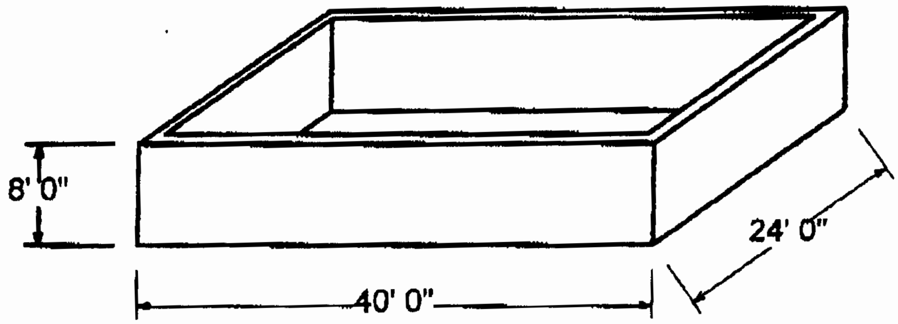
22. Volume =



7' - 0" Radius

Carpentry

23. A concrete foundation 8'-0" deep measures 24'-0" by 40-0"' (outside dimensions) and has walls 9" thick. How many cubic yards of concrete are there in the foundation walls?



24. Solve by calculator. Give your answer to the nearest hundredth:

$\sqrt{89}$

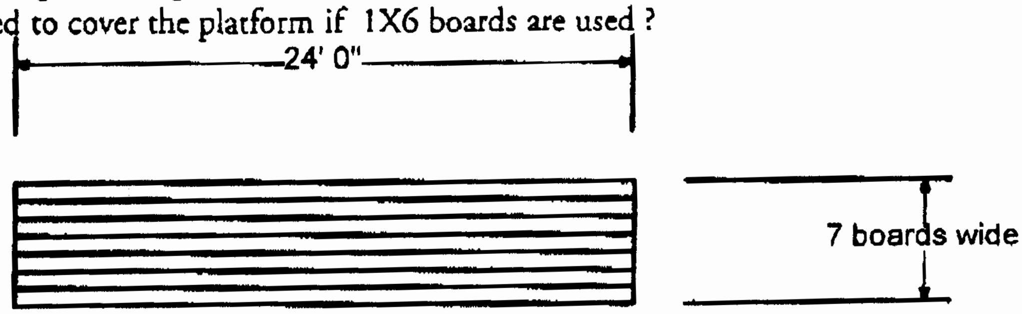
25a. Find the number of board feet in 84 pieces of 2 x 10 x 14'.

25b. Find the number of linear feet of 1 x 8 boards needed to cover a deck measuring 14'-0" x 20'-0". Do not add in extra for error and waste.

26a. How many cubic yards of concrete is needed for the foundation footing of a house with dimensions 26'-0" x 30'-0" to the outside of the foundation wall. The six inch wall is centered on the footing. The footing is to be 20 inches wide and 9 inches thick.

26b. Find the cubic yards of concrete needed for a cellar floor in a 28'-0" x 32'-0" house. The slab is to be 3 inches thick.

27. Rough flooring: A platform is shown below. How many board feet of subflooring are needed to cover the platform if 1X6 boards are used ?



28. Finished Flooring: How many square feet of strip flooring are needed to cover a first floor plan which measures 24' —0" x 30' – 0" ?

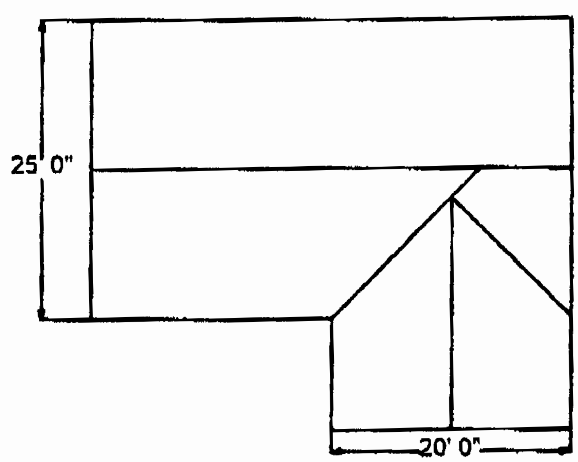
29. Find the length of a common rafter if the span is 28'- 0", total rise is 7'-0" and the projection is 1' – 6".

30. A roof has a 5/12 slope. If an 18 inch horizontal projection of the overhang is desired, what will be the overhang length?

31. The girder span in a house is 29'-0". How many board feet of a solid 6" x 8" girder is required?

32a. Find the length of a hip rafter if the common rafter measures 12 ft 7 13/16 inches and the run of the common rafter measures 12 ft-0".

32b. Suppose the main building is 25'-0" wide and the ell is 20'-0". wide. Both roofs have a 5/12 slope. Find the lengths of both the long (1) and (2) short valley rafters.



34. How many bundles of 18 – inch wood shingles laid 6 inches to the weather are required for one sideof a gable roof measuring 16'-0" x 32'-0"? There are 4 bundles per square. (Allow 8% for waste).

35. Using the chart below, find how many pounds of 6d siding nails are needed to put up 3500 board feet of siding.

QUANTITIES OF NAILS REQUIRED

Materials to be fastened	Size and kind	Pounds needed	Unit
Joists and sills	16d common	20	1000 bd ft
Studs, rafters	8d, 16common	20, 25	1000 bd ft
Composition Shingles	1 1/2 " galv.	3	square
Sheathing	8d common	20	1000 bd ft
Siding	6d siding	18	1000 bd ft
Flooring	8d casing	30	1000 bd ft
Wood Shingles	8d galv.	5 1/3	square

36a. Find the number of risers (1) and (2) unit rise for a set of stairs having a total rise of 98 inches.

36b. Find the total run of a set of stairs having an 11 ¼ inch unit run and 98 inches of total rise.

INTRODUCTION TO RATIO AND PROPORTION

Problem solving through the utilization of **RATIO** and **PROPORTION**, has extensive practical application within the workplace and in many technical fields of study. If you think about it for a minute, RATIO and PROPORTION are involved in our lives in more ways than we consciously realize. The mixing of ingredients, reading scaled drawings, sizing pulleys and gears, determining transformer voltages and electrical current ratios, are just a few examples of how our lives are filled with the unique relationships defined by RATIO and PROPORTION.

In this Chapter, we will learn what a RATIO is, how to properly set one up, and then the proper way to use ratios in establishing **PROPORTIONAL STATEMENTS**. Solutions for both **DIRECT** and **INVERSE PROPORTIONS** will also be presented and discussed. And an old friend, the word problem, will be examined to see how it relates to RATIO and PROPORTION. As you may have already experienced, day-to-day activities often make it necessary for us to develop correct STATEMENTS OF PROPORTION that are based on verbal, written, and/or graphic information.

RATIOS and resulting PROPORTIONS are fundamentally based, mathematically, on **FRACTIONAL RELATIONSHIPS**. Your knowledge and previous study in this area will be very beneficial to you, as you develop an understanding of, and competency in, solving problems using the principles of RATIO and PROPORTION.

RATIOS

A **RATIO** is a divisionary comparison of two **LIKE QUANTITIES** that are *both* expressed in the **SAME UNITS OF MEASUREMENT.** Therefore, by definition, a RATIO does in fact take on the form of a fraction.

THE ORDER IN WHICH A RATIO IS WRITTEN MUST CORRESPOND PROPERLY WITH THE WAY THE PROBLEM IS STATED.

In other words, the way the quantities are placed in the NUMERATOR and the DENOMINATOR is **crucial.** If this rule is not respected, you'll end up with some bizarre and *extremely* incorrect relationships.

IN DETERMINING A RATIO, THE FINAL ANSWER DOES NOT CONTAIN UNIT IDENTIFICATION, IT IS SIMPLY EXPRESSED AS A STANDARD RATIO STATEMENT.

The answers in the following examples will illustrate this principle.

EXAMPLE.

*What is the **ratio** of **2 feet** to **6 inches**?*

*First, we need to determine if a **RATIO** is indeed possible for this relationship. Are the **quantities** being compared **alike**?. . . If so, are they **expressed** in the **same units of measure**?. . . In this problem, the quantities involved **are** alike, they are both LENGTHS. Therefore, we should be able to establish a mathematical "RATIO" between them. To set the RATIO up properly, the **FIRST QUANTITY** stated (**2 feet**), should be placed in the NUMERATOR, and the **SECOND QUANTITY** stated (**6 inches**), should be placed in the DENOMINATOR.*

$$\frac{\text{2 feet}}{\text{6 inches}}$$ ☜ **1ST QUANTITY** ☜ **2ND QUANTITY**

*Well, there's our RATIO . . . or is it? Both measurements are for length, but the units of measurement are **not** the same; one is in **feet** and one is in **inches**. Our basic rule for ratios stated that the units of measurement had to be the **same**, therefore, a unit conversion is necessary in order to express both quantities in the **same** units. In this case, we will convert feet to inches.*

$$2\ \cancel{\text{feet}} \times \frac{12\ \text{inches}}{1\ \cancel{\text{foot}}} = 24\ \text{inches}$$

*Substituting inches for feet, we now have **LIKE QUANTITIES** expressed in LIKE UNITS. The resulting **ratio** is:*

$$\frac{\text{24 inches}}{\text{6 inches}}$$ ☜ **1ST QUANTITY** ☜ **2ND QUANTITY**

*The fraction 24/6 is now reduced to its **LOWEST TERMS** and the units of measure will cancel each other out.*

$$\frac{\text{24 inches}}{\text{6 inches}} = \frac{4}{1}$$ ☜ **1ST QUANTITY** ☜ **2ND QUANTITY**

or

1ST QUANTITY ☞ 4 to 1 ☜ **2ND QUANTITY**

The **final form** of a RATIO is *never* left as a FRACTION . Instead, it is written as a **STATEMENT OF THE RATIO RELATIONSHIP.** Subsequently, the fraction **4/1**, in this problem, would be written as **4 to 1**. In some cases, you will find that the word "**to**" is replaced by a colon **(:)** when expressing ratios. For example, **4 to 1** could just as well be written as **4:1**. Both forms are quite acceptable; no matter which you use, the ratio is exactly the same and should be read as: **four to one**.

The RATIO of **2 feet** to **6 inches** is **4 to 1**.

EXAMPLE.

*What is the RATIO of **6 inches** to **2 feet**?*

Notice any similarities to the previous problem? . . . That's right, this problem has the ***same quantities*** *that were used in the previous example, with one major exception,* ***the RATIO is reversed****. And, as we have just studied, the way this problem is stated, the* ***RATIO*** *should be written with* ***6 inches*** *in the NUMERATOR and* ***2 feet*** *in the DENOMINATOR . Remember, the ratio rules stipulated that* ***the order in which the ratio is written*** *is crucial to correctly defining a ratio relationship.*

$$\frac{6\text{ inches}}{2\text{ feet}}$$ ☜ **THE RATIO PROPERLY WRITTEN**

$$\frac{6\text{ inches}}{24\text{ inches}}$$ ☜ **FEET CONVERTED TO INCHES, UNITS MUST BE THE SAME**

$$\frac{1}{4}$$ ☜ **FRACTION REDUCED WITH UNITS CANCELING**

This result, would be properly written as a ratio of: **1 to 4** or **1 : 4**. It would be stated as: The RATIO of **6 inches** to **2 feet** is **one to four**.

EXAMPLE.

What is the RATIO of ***3 hours*** *to* ***15 minutes****?*

First things first, are the quantities stated ***alike****? . . . Yes they are, both of them are measurements of time. Therefore, a ratio relationship is possible, let's proceed.*

$$\frac{3\text{ hours}}{15\text{ min.}}$$ ☜ **1ST QUANTITY** ☜ **2ND QUANTITY**

Convert 3 hours to minutes.

$$3\ \cancel{\text{hours}} \times \frac{60\text{ min.}}{1\ \cancel{\text{hour}}} = 180\text{ min.}$$

$$\frac{180\ \cancel{\text{min.}}}{15\ \cancel{\text{min.}}}$$ ☜ **1ST QUANTITY (same units)** ☜ **2ND QUANTITY (same units)**

SIMPLIFIED ☞ $$\frac{180\ \cancel{\text{min.}}}{15\ \cancel{\text{min.}}} = \frac{12}{1}$$ ☜ **1ST QUANTITY** ☜ **2ND QUANTITY**

The RATIO is:

12 to **1**

Could we have converted to "like units" using HOURS instead of MINUTES ? . . . Absolutely. Here is how that solution would look.

$$15\ \cancel{\text{min.}} \times \frac{1\text{ hour}}{60\ \cancel{\text{min.}}} = \frac{15}{60}\text{ hour} = \frac{1}{4}\text{ hour}$$

$$\frac{3 \text{ hours}}{\frac{1}{4} \text{ hours}}$$ ☜ 1^{ST} QUANTITY ☜ 2^{ND} QUANTITY

Simplifying the ratio.

$$3 \cancel{\text{hours}} \times \frac{4}{1 \cancel{\text{hours}}} = \frac{12}{1}$$ ☜ 1^{ST} QUANTITY ☜ 2^{ND} QUANTITY

The RATIO is:

12 to 1

You will notice that the **RATIO** is still the same, no matter which unit of measurement we choose to make the "common" unit; and logically thinking, that makes sense. If a UNIT CONVERSION is done correctly, and a ratio relationship does in fact exist, the unit that is chosen for computation of the ratio should have absolutely no bearing whatsoever on the outcome of that computation. That is, THE UNIT OF MEASUREMENT SELECTED WILL HAVE NO EFFECT ON THE RATIO THAT EXISTS BETWEEN TWO LIKE QUANTITIES . Naturally, you should choose a conversion that is easy to work with and that works well with your ratio computation. Once again, you need to look a little bit ahead in your computation, before you can actually decide which unit would be best to use. In this case, converting hours to minutes was the more logical choice, as it resulted in a **simpler** conversion and subsequent ratio computation.

EXERCISE 10-1.

Determine the following RATIOS and express them in their proper form. Think about the conversions that are involved before you actually set the ratio computations up.

1) *What is the ratio of 6 ounces to 3 pounds?*
2) *What is the ratio of 24 volts to 3 volts?*
3) *What is the ratio of 3 yards to 12 inches?*
4) *What is the ratio of 4 hours to 2 days?*

PROPORTION

What is a **PROPORTION** and how is it connected to RATIOS ?

PROPORTION IS A STATEMENT WHICH STATES THAT TWO RATIOS ARE EQUAL TO EACH OTHER. IT CAN BE EXPRESSED AS A SENTENCE OR, MORE COMMONLY, AS AN ALGEBRAIC EQUATION THAT PROPERLY REPRESENTS THE RELATIONSHIPS STATED.

Remembering that RATIOS are mathematically expressed as fractions, it would stand to reason then; that a **PROPORTION**, mathematically stated, would look like **two** fractions with an **equals** sign in between them.

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}$$ ☜ **A PROPORTION**

The preceding proportion states that:

$$\text{The RATIO } \frac{A_1}{A_2} \textbf{ is equal to } \text{the ratio } \frac{B_1}{B_2}$$

Therefore, EQUIVALENT FRACTIONS could be stated as a PROPORTIONAL RELATIONSHIP.

$$\frac{3}{4} = \frac{6}{8}$$ **A PROPORTION**

Why does this work out this way? . . . Because, by definition, a **PROPORTION** is nothing more than a stated relationship between to ratios (fractions) that are **equal** in value.

In a problem that involves PROPORTION; generally, **one** of the **four quantities** involved in the two ratios is UNKNOWN, and the mathematical challenge is to determine what that unknown quantity actually is.

Consider our previous example of equivalent fractions (3/4 = 6/8) that actually forms a proportional statement. If the DENOMINATOR of the second ratio (6/8) were UNKNOWN, and we knew for fact that a proportional situation did exist, the proportion would be written as **3/4 = 6/X.** Stated this way, we could then proceed to solve for **X**, the second ratio's DENOMINATOR .

The unknown quantity in a PROPORTION statement is determined by a method known as **CROSS MULTIPLICATION**. Simply stated, CROSS MULTIPLICATION is the process of multiplying the **NUMERATOR** of one ratio by the **DENOMINATOR** of the other ratio. The resulting answer is known as the **CROSS PRODUCT**. The CROSS PRODUCTS of a proportion have a unique relationship.

THE CROSS PRODUCTS OF THE EQUIVALENT FRACTIONS IN A PROPORTION ARE ALWAYS EQUAL. THEREFORE, IN A PROPORTION, THE 1ST CROSS PRODUCT IS EQUAL TO THE 2ND CROSS PRODUCT.

In our example, let's CROSS MULTIPLY the **numerator** of the **first ratio (3)** by the **denominator** of the **second ratio (X)**.

$$\frac{3}{4} = \frac{6}{X}$$

3 times **X = 3X** **1ST CROSS PRODUCT**

Now, to determine what our **SECOND CROSS PRODUCT** is, we need to *multiply* the **denominator** of the **first ratio (4)** by the **numerator** of the **second ratio (6)**.

$$\frac{3}{4} = \frac{6}{X}$$

4 times **6 = 24** **2ND CROSS PRODUCT**

Equating the two cross products, we have:

Now, by using traditional Algebraic procedures, we can solve for **X** and determine what the proportional UNKNOWN quantity is.

$$\frac{3X}{3} = \frac{24}{3}$$

$$X = 8$$

Let's take a moment to evaluate this **cross product** procedure and see if we can determine just why it actually works.

$\frac{3}{4} = \frac{6}{8}$ implies that $\frac{3}{4} \div \frac{6}{8}$ is equal to 1, because anything divided by itself, or something equal to itself, ***always*** equals **1**.

$$\frac{3}{4} \div \frac{6}{8} = \frac{3}{4} \times \frac{8}{6} = \frac{24}{24} = \frac{1}{1} = \mathbf{1}$$

Notice how this "proving process" included multiplying 3 × 8 and 4 × 6. In both situations, naturally, the resulting product was equal to **24**. What we have actually represented, are the CROSS PRODUCTS of the original proportion statement. And, as we studied, **they must always be equal to each other.** This fact allows us to solve proportion problems by setting the cross products **equal to** each other, and then solving the resulting equation to determine the unknown variable.

Hopefully, this little "proving exercise" has helped you understand how important it is to know why a particular rule works, as well as how it works. This **adds** tremendously to your understanding of mathematics and gives you confidence in your problem solving ability. Remember, it's one thing to memorize and use a mathematical rule, and something quite different to **understand** exactly why that rule works.

The following are examples of how to solve PROPORTION STATEMENTS using CROSS MULTIPLICATION.

EXAMPLE.

Solve the PROPORTION.

1ST RATIO ☞ $\frac{6}{9} = \frac{2}{A}$ ☜ **2ND RATIO**

Cross multiply the numerators and the denominators and set the cross products equal to each other.

$$6 \times A = 9 \times 2$$

1ST CROSS PRODUCT ☞ **6A = 18** ☜ **2ND CROSS PRODUCT**

Now, solve the resulting Algebraic equation.

$$\frac{6A}{6} = \frac{18}{6}$$

The unknown variable in the proportion is **3**.

To verify our results, we use the **SUBSTITUTION METHOD**.

$$\frac{6}{9} = \frac{2}{A}$$

$$\frac{6}{9} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3}$$

EXAMPLE.

Solve the PROPORTION.

$$\frac{2}{5} = \frac{X}{15}$$

CROSS MULTIPLY.

$$2 \bullet 15 = 5 \bullet X$$

$$30 = 5X$$

Solve the resulting EQUATION.

$$\frac{5X}{5} = \frac{30}{5}$$

$$X = 6$$

The unknown variable in this proportion is 6.

Substituting to ***VERIFY.***

$$\frac{2}{5} = \frac{X}{15}$$

$$\frac{2}{5} = \frac{6}{15}$$

$$\frac{2}{5} = \frac{2}{5}$$

EXAMPLE.

Solve the PROPORTION.

$$\frac{R}{10} = \frac{3}{30}$$

Determine the CROSS PRODUCTS.

$$R \bullet 30 = 10 \bullet 3$$

Solve the EQUATION.

$$\frac{30R}{30} = \frac{30}{30}$$

$$R = 1$$

VERIFY *using substitution.*

$$\frac{R}{10} = \frac{3}{30}$$

$$\frac{1}{10} = \frac{3}{30}$$

$$\frac{1}{10} = \frac{1}{10}$$

EXAMPLE.

Solve the PROPORTION.

$$\frac{12}{C} = \frac{18}{6}$$

CROSS MULTIPLY.

$$12 \bullet 6 = C \bullet 18$$

$$72 = 18C$$

$$18C = 72$$

Solve the EQUATION.

$$\frac{18C}{18} = \frac{72}{18}$$

$$C = 4$$

VERIFYING *through substitution.*

$$\frac{12}{C} = \frac{18}{6}$$

$$\frac{12}{4} = \frac{18}{6}$$

$$3 = 3$$

As was illustrated in the preceding examples, the UNKNOWN VARIABLE can appear in either the **numerator** or the **denominator**, of either one of the RATIOS (fractions) involved in the **PROPORTIONAL RELATIONSHIP**. Regardless of **where** the variable appears, the cross multiplication method will *always* result in the correct solution.

The ability to properly write RATIOS and solve PROPORTION STATEMENTS is something that is extremely useful in all walks-of-life. Ratios and corresponding proportions are everywhere, have fun solving them and enjoy putting to use the information that they provide to you.

EXERCISE 10-2.

Solve the following PROPORTION STATEMENTS for the unknown variable using the cross multiplication method.

1) $\frac{A}{3} = \frac{18}{9}$ 2) $\frac{2}{B} = \frac{4}{8}$

3) $\frac{5}{2} = \frac{C}{6}$ 4) $\frac{2}{3} = \frac{8}{D}$

DIRECT PROPORTION

Many problems can be solved by writing a **PROPORTION STATEMENT** that involves **four** quantities (two ratios) which are related through a proportional relationship. HOW the quantities are related, *will determine* the manner in which the proportion is written. How? . . . you might ask . . . Yes, how??? . . . For, if you think about it for a minute, given what we know about PROPORTIONS, they actually could, from a mathematical perspective, be written to represent a **DIRECT** or an **INDIRECT** proportional equality. This section of the text, will explore problems that involve quantities which are **DIRECTLY PROPORTIONAL** to each other.

Two quantities are DIRECTLY PROPORTIONAL to each other if an *increase* in one quantity results in a **corresponding and proportional** *increase* in the other quantity. Furthermore, two quantities are also DIRECTLY PROPORTIONAL if a decrease in one quantity results in a **corresponding and proportional** *decrease* in the other quantity. In other words, if **one thing happens**, then **the other thing happens**; in the **same** direction and at the **same** exact proportional rate.

How about some examples of quantities that are directly proportional? . . . We deal with them every day.

Rate (or speed) and distance are DIRECTLY PROPORTIONAL. The **faster** you travel, in a given *amount* of time, the *farther* you will travel. Additionally, if you *decrease* your speed, for the same amount of time, you will cover *less* distance. In other words, for the same amount of time, **increased** speed results in **increased** distance traveled; and **decreased** speed, results in **decreased** distance traveled-proportionally.

Here's another. The relationship of the number of items purchased (at the same unit cost), and the total cost of those items, is DIRECTLY PROPORTIONAL. Consequently, if you buy more items, your total cost increases, **proportionally**. If you buy fewer items, your total cost decreases, **proportionally**. In essence, that is what DIRECT PROPORTION is all about; nothing more, nothing less.

THE QUANTITIES STATED IN A DIRECT PROPORTION ARE DIRECTLY RELATED. AS ONE GOES UP, SO DOES THE OTHER ONE – IN A PROPORTIONAL MANNER. THE SAME HOLDS TRUE FOR MOVEMENT IN THE OPPOSITE DIRECTION.

SOLVING DIRECT PROPORTION PROBLEMS

Here is a step-by-step procedure for you to follow when you are solving problems that deal with a DIRECT PROPORTION relationship. Remember, the **most important** part of the problem is the **set-up**; if it is done incorrectly, it is impossible to obtain an accurate result.

STEP 1. ***Determine the VARIABLES*** *that are involved in the problem and choose* ***appropriate*** *letters to represent them.*

STEP 2. *Write a RATIO that represents one of the VARIABLES, the* ***order*** *at this point is immaterial—simply make note of whatever order you decide upon.*

STEP 3. *Write a RATIO for the remaining VARIABLE. At this point, the* ***order*** *that you use for the SECOND RATIO is* ***extremely*** *important, as it* ***must*** *match the* ***order*** *that you established in the preceding step.*

STEP 4. *Write a PROPORTION STATEMENT by setting the two ratios* ***equal*** *to each other, respecting the order that has been established.*

STEP 5. *Substitute the KNOWN QUANTITIES into the equation and* ***solve*** *for the UNKNOWN QUANTITY using CROSS MULTIPLICATION.*

EXAMPLE.

If three gears, of equal weight, weighed a total ***54*** *pounds, how much would seven of these gears weigh?*

STEP 1. *The variables are the NUMBER OF GEARS and the WEIGHT. Let's have* ***N*** *stand for the number of gears, and* ***W*** *for the weight of those gears. Therefore, our variables will be represented by* ***N*** *and* ***W****.*

We actually need to represent ***four*** *different quantities that are involved in this PROPORTION. This will amount to two different numbers for the "number of gears", and two different numbers for their "weights". Therefore, suitable notations for this particular problem would be:* $\mathbf{N_1}$, $\mathbf{N_2}$ *and* $\mathbf{W_1}$, $\mathbf{W_2}$. *With this type of representation, the* ***subscripts*** *become extremely important, as they allow us to identify which of the variables we are referring to. In this example, the* ***three gears*** *mentioned weigh* ***54*** *pounds. It would make sense then to Let* N_1 *= THREE GEARS and* W_1 *= 54 POUNDS. Now, since* ***seven gears****, that are identical to the first three gears, have an* ***unknown weight****, it would follow that we should let* N_2 *= SEVEN GEARS and* W_2 *= the UNKNOWN WEIGHT.*

STEP 2. *Choose either variable (number of gears (N), or weight of gears (W)), and establish a RATIO for it. In this case, let's write a ratio for the NUMBER OF GEARS (N).*

$$\frac{N_1}{N_2}$$

You will notice that this ratio is made up of two like quantities; they both represent a certain NUMBER OF GEARS.

STEP 3. *Now we need to write another ratio, this time for WEIGHT. And please note, it must be established in the* ***same order*** *in which the first one was written. Therefore, since* $\boldsymbol{N_1}$ *was in the* ***numerator*** *of the* ***first ratio,*** $\boldsymbol{W_1}$ *has to be in the* ***numerator*** *of the* ***second ratio****. Furthermore,* $\boldsymbol{N_2}$ *was in the* ***denominator*** *of the* ***first ratio****, consequently,* $\boldsymbol{W_2}$ *will appear in the* ***denominator*** *of the* ***second ratio****—the order must be maintained. Here's what the second ratio, which is for weight, will look like.*

$$\frac{W_1}{W_2}$$

STEP 4. *With the two ratios established, we can now write them set up to equal each other, which results in the following PROPORTION STATEMENT.*

$$\frac{N_1}{N_2} = \frac{W_1}{W_2}$$

Step 5. *Substitute the KNOWN VALUES into the equation and solve for the remaining UNKNOWN by using cross multiplication.*

$$\frac{N_1}{N_2} = \frac{W_1}{W_2}$$

$$\frac{3\ \cancel{\text{gears}}}{7\ \cancel{\text{gears}}} = \frac{54\ \text{pounds}}{W_2}$$

$$3W_2 = 378\ \text{pounds}$$

$$W_2 = 126\ \text{pounds}$$

Answer: The **7** gears in question weigh a total of **126** pounds.

To VERIFY our results, we will substitute ALL known quantities back into the original proportional equation.

$$\frac{N_1}{N_2} = \frac{W_1}{W_2}$$

$$\frac{3}{7} = \frac{54}{126}$$

$$\boldsymbol{\frac{3}{7} = \frac{3}{7}}$$

In the previous example, we used a word problem to illustrate each step of the solution in clear detail. The remaining examples in this Section will be presented in a more generic format, with the KNOWN variables already assigned. In other words, **STEP 1** of the solution process just presented, will not be necessary; the appropriate designations will already be established. Fear not, you will get plenty of additional practice doing **STEP 1** when we consider word problems, later in this Chapter. For now, let's concentrate on **STEPS 2, 3, 4** and **5**, and become thoroughly familiar with solving DIRECT PROPORTION problems; with the variables already given. You will also notice that specific UNITS OF MEASUREMENT are **not** assigned in these general examples.

EXAMPLE.

A *and* ***B*** *are DIRECTLY proportional.*

If $A_1 = 15$, $A_2 = 6$, and $B_1 = 90$, determine the value of B_2.

Select either variable and make a ratio of it. Once again, when establishing the ***first ratio****, the order is not important.*

$$\frac{A_1}{A_2}$$

Now write a ratio for the second variable, and remember, it must be in the same order as the first ratio. This is a DIRECT PROPORTION; therefore, the ***order*** *of the two ratios must be the* ***same****.*

$$\frac{B_1}{B_2}$$

A good way to check to be sure that your order is correct, is to evaluate the positions of A_1 in the FIRST RATIO and B_1 in the SECOND RATIO. If they are in the same fractional positions (numerator-to-numerator or denominator-to-denominator) then your order is correct.

Now the ratios are set equal to each other, forming a DIRECT PROPORTION STATEMENT.

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}$$

The KNOWN VALUES are substituted into the proportion.

$$\frac{15}{6} = \frac{90}{B_2}$$

Determine the CROSS PRODUCTS by performing CROSS MULTIPLICATION.

$$15B_2 = 540$$

Solve the resulting equation to determine the remaining UNKNOWN VALUE.

$$B_2 = 36$$

VERIFY the results by substituting the values back into the original proportion statement.

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}$$

$$\frac{\mathbf{15}}{\mathbf{6}} = \frac{\mathbf{90}}{\mathbf{36}}$$

Reduce each fraction (ratio) to its lowest terms.

$$\frac{\mathbf{5}}{\mathbf{2}} = \frac{\mathbf{5}}{\mathbf{2}}$$

By now you have to be wondering . . . "Couldn't that problem be set up and solved another way?". . . It sure could! Either variable could have been chosen for the FIRST RATIO-and-the FIRST RATIO could have been written in any order. Think about it for a minute . . . How many different ratios could have been written as the "first ratio" in this problem? . . . ? That's right, ***four****! Let's write them out.*

$$\frac{A_1}{A_2} \text{ or } \frac{A_2}{A_1} \text{ or } \frac{B_1}{B_2} \text{ or } \frac{B_2}{B_1}$$

All of the above ratios would be acceptable. As we have discovered, the real *critical* step in solving DIRECT PROPORTION problems is in how we write the SECOND RATIO—**it must be in the same order as the first.** This fact cannot be overemphasized. If you are going to work with DIRECT PROPORTION, you must understand this concept. The following chart provides a graphic illustration of the parameters that are involved in this relationship.

FIRST RATIO = SECOND RATIO (SAME ORDER)		**CROSS PRODUCTS**
I.	$\frac{A_1}{A_2} = \frac{B_1}{B_2}$	$A_1B_2 = A_2B_1$
II.	$\frac{A_2}{A_1} = \frac{B_2}{B_1}$	$A_2B_1 = A_1B_2$
III.	$\frac{B_1}{B_2} = \frac{A_1}{A_2}$	$B_1A_2 = B_2A_1$
IV.	$\frac{B_2}{B_1} = \frac{A_2}{A_1}$	$B_2A_1 = B_1A_2$

Notice that the resulting CROSS PRODUCTS are **identical** in each case, and they will subsequently produce **identical solutions**. Let's prove it to ourselves by taking each possible CROSS PRODUCT equation and substituting the original values of $A_1 = 15$, $A_2 = 6$, $B_1 = 90$ into it, and then proceed to solve for B_2

I. $A_1B_2 = A_2B_1$

$$15B_2 = 6 \bullet 90$$

$$15B_2 = 540$$

$$B_2 = 36$$

II. $A_2B_1 = A_1B_2$

$$6 \bullet 90 = 15B_2$$

$$540 = 15B_2$$

$$15B_2 = 540$$

$$B_2 = 36$$

III. $B_1A_2 = B_2A_1$

$$90 \bullet 6 = B_2 \bullet 15$$

$$540 = 15B_2$$

$$B_2 = 36$$

IV. $B_2A_1 = B_1A_2$

$$B_2 \bullet 15 = 90 \bullet 6$$

$$15B_2 = 540$$

$$B_2 = 36$$

Any difference in our solutions? . . . No, there sure isn't; and there shouldn't be. The correct answer for B_2 can be determined using *any* of the four different approaches. The **key** is, to *always* set up the **SECOND RATIO** in the same order as the **FIRST**.

EXAMPLE.

***B** and **D** are DIRECTLY PROPORTIONAL. If $B_1 = 27$, $D_1 = 3$ and $D_2 = 8$, determine what the value of $\boldsymbol{B_2}$ is.*

Establish the FIRST RATIO.

$$\frac{D_2}{D_1}$$ ☜ **ANY VARIABLE—ANY ORDER**

Establish the SECOND RATIO.

$$\frac{B_2}{B_1}$$ ☜ **SAME ORDER AS FIRST RATIO**

Set the ratios equal to each other to establish a PROPORTION STATEMENT.

$$\frac{D_2}{D_1} = \frac{B_2}{B_1}$$ ☜ **ORDER IS MAINTAINED**

Substitute the known values into the proportion.

$$\frac{8}{3} = \frac{B_2}{27}$$

Determine the CROSS PRODUCTS by using cross multiplication.

$$3B_2 = 8 \bullet 27$$

$$3B_2 = 216$$

Solve the resulting equation to determine the remaining UNKNOWN VALUE.

$$B_2 = \frac{216}{3}$$

$$\mathbf{B_2 = 72}$$

Verify the results by substituting the values back into the ***original*** *proportion statement.*

$$\frac{D_2}{D_1} = \frac{B_2}{B_1}$$

$$\frac{8}{3} = \frac{72}{27}$$

$$\frac{8}{3} = \frac{8}{3}$$

EXAMPLE.

D *and* ***T*** *are DIRECTLY proportional.*

Find the value of $\boldsymbol{T_2}$, *if* $D_1 = 280$, $D_2 = 200$ *and* $T_1 = 7$.

FIRST RATIO.

$$\frac{T_1}{T_2}$$

SECOND RATIO.

$$\frac{D_1}{D_2}$$

PROPORTION STATEMENT.

$$\frac{T_1}{T_2} = \frac{D_1}{D_2}$$

VALUE SUBSTITUTION.

$$\frac{7}{T_2} = \frac{280}{200}$$

CROSS MULTIPLICATION.

$$280T_2 = 7 \bullet 200$$

$$280T_2 = 1400$$

EQUATION SOLVING.

$$280T_2 = 1400$$

$$T_2 = \frac{1400}{280}$$

$$\mathbf{T_2 = 5}$$

VERIFY USING SUBSTITUTION.

$$\frac{T_1}{T_2} = \frac{D_1}{D_2}$$

$$\frac{7}{5} = \frac{280}{280}$$

$$\frac{\mathbf{7}}{\mathbf{5}} = \frac{\mathbf{7}}{\mathbf{5}}$$

Naturally, a calculator may be used for the intermediate mathematical steps involved in solving PROPORTIONAL STATEMENTS.

EXERCISE 10-3.

Solve the following problems that involve DIRECT proportions. Verify your answers when you have determined what the UNKNOWN value is.

1) *A and B are directly proportional. Find* A_1, *if* $A_2 = 56$, $B_1 = 4$ *and* $B_2 = 7$.

2) *L and M are directly proportional. Find* M_1, *if* $L_1 = 35$, $L_2 = 45$ *and* $M_2 = 63$.

3) *R and V are directly proportional. Find* V_2, *if* $R_1 = 20$, $R_2 = 35$ *and* $V_1 = 60$.

INVERSE PROPORTION

Two quantities are **INVERSELY PROPORTIONAL** to each other if an *increase* in the FIRST QUANTITY produces a **corresponding** and **proportional** *decrease* in the SECOND QUANTITY; or if a *decrease* in the FIRST QUANTITY results in a **corresponding** and **proportional** *increase* in the SECOND QUANTITY. In other words, there is an absolute **inverse** relationship between the two quantities involved, if one goes **up** . . . the other goes **down**; if one goes **down** . . . the other goes **up**-on a PROPORTIONAL BASIS that can be represented as a FRACTIONAL EQUATION.

As with **DIRECT** proportions, **INVERSE** proportions are all around us and it is essential that we understand what they actually represent and how to work with them. Let's examine a couple of common examples that illustrate quantities which are INVERSELY PROPORTIONAL.

Assuming that the same exact distance is to be traveled, the rate (or speed) of travel is INVERSELY PROPORTIONAL to the time actually required to travel that distance. Simply put, if we travel *faster* (our rate is **increased**), it will take us *less* time (our time is **decreased**) to travel the same distance. Conversely, by **decreasing** our rate of travel, we would actually **increase** the amount of time required to travel the same distance. Therefore, in this situation we can say that RATE and TIME are **inversely proportional**.

Another common example involves pulleys and their rate of revolution. The SPEED of a driven pulley (measured in RPM) is INVERSELY proportional to its DIAMETER. That is, if the DIAMETER of a pulley **increases**, its actual RATE of rotation (RPM) will **decrease**. Conversely, if the diameter is **decreased**, the rate of revolution will **increase**. Therefore, SPEED and DIAMETER are **inversely proportional**.

10.7 SOLVING INVERSE PROPORTION PROBLEMS

The methodology used to solve problems which involve **INVERSE PROPORTIONS** is very similar to that which is used to solve DIRECT proportion problems. In fact, the steps are **identical**, except for **STEP 3**; WRITING THE SECOND RATIO. In an **inverse** proportion problem, the SECOND RATIO must be written in the **opposite order** of the FIRST RATIO. Here again, this particular step is *crucial* to the problem solving procedure. The SECOND RATIO *must be* written in the opposite order of the FIRST RATIO. Let's look at a few examples of how this **inverse** writing of the SECOND RATIO should actually appear. We will use the generic values **A** and **B** for illustrative purposes.

FIRST RATIO = SECOND RATIO
(OPPOSITE ORDER)

I. $\frac{A_1}{A_2} = \frac{B_2}{B_1}$

II. $\frac{A_2}{A_1} = \frac{B_1}{B_2}$

III. $\frac{B_1}{B_2} = \frac{A_2}{A_1}$

IV. $\frac{B_2}{B_1} = \frac{A_1}{A_2}$

As you can see, the SECOND RATIO is always in the **inverse** order of the FIRST RATIO. A good way to ensure that you have set up an INVERSE PROPORTION correctly, is to check for the positions occupied in the **numerators** and **denominators** of your ratio equations. For instance, if the value $\mathbf{A_1}$ appears in the NUMERATOR of the **first ratio**, then its corresponding value $\mathbf{B_1}$, **must** appear in the DENOMINATOR of the **second ratio**. And, as you might expect, if a variable value appears in the DENOMINATOR of the **first ratio**, then its corresponding variable value **must** be written in the NUMERATOR of the **second ratio**. INVERSE PROPORTION and OPPOSITE ORDER are as simple as that. Once you have written the second ratio properly (in the *opposite* order of the first ratio), you simply set the two ratios equal to each other and determine their **CROSS PRODUCTS** by using CROSS MULTIPLICATION; sound familiar? Next, solve the resulting equation and you have identified your UNKNOWN value. To verify, plug **all** of your values back into the original INVERSE PROPORTION equation, and see if you do, in fact, have "balance".

Let's walk through a few examples illustrating (step-by-step) the procedures we just reviewed for solving INVERSE PROPORTION problems. Our examples will already have their variables assigned and units of measurement will not be identified.

EXAMPLE.

***A** and **B** are INVERSELY proportional.*

If $A_1 = 1$, $A_2 = 3$ and $B_1 = 15$, determine the value of $\mathbf{B_2}$.

Decide which VARIABLE you want to start with (either one, it makes no difference) and establish the FIRST RATIO, in any order you choose to use (once again, it makes no difference).

$$\frac{A_1}{A_2}$$

*Now write a ratio for the SECOND VARIABLE, and remember, it must be in the **opposite** order of the first ratio. This is an INVERSE PROPORTION; therefore, the **order** of the two ratios must be **opposite**.*

$$\frac{B_2}{B_1}$$

*A good way to check to be sure that your order is correct, is to evaluate the positions of $\mathbf{A_1}$ in the first ratio and $\mathbf{B_1}$ in the second ratio. If they are in the **opposite fractional positions** (numerator-to-denominator or denominator-to-numerator) then your order is correct.*

Now the ratios are set equal to each other, forming an INVERSE PROPORTION STATEMENT.

$$\frac{A_1}{A_2} = \frac{B_2}{B_1}$$

The KNOWN VALUES are substituted into the proportion.

$$\frac{1}{3} = \frac{B_2}{15}$$

Determine the CROSS PRODUCTS by performing CROSS MULTIPLICATION.

$$3B_2 = 15$$

Solve the resulting equation to determine the remaining UNKNOWN VALUE.

$$B_2 = \frac{15}{3}$$

$$\mathbf{B_2 = 5}$$

Verify the results by substituting all identified values back into the original proportion statement.

$$\frac{A_1}{A_2} = \frac{B_2}{B_1}$$

$$\frac{1}{3} = \frac{5}{15}$$

$$\mathbf{\frac{1}{3} = \frac{1}{3}}$$

EXAMPLE.

***C** and **D** are INVERSELY proportional.*

Determine the value of $\mathbf{C_1}$, if $C_2 = 2$, $D_1 = 8$ and $D_2 = 12$.

Establish the FIRST RATIO.

$$\frac{C_1}{C_2}$$ **ANY VARIABLE—ANY ORDER**

Establish the SECOND RATIO.

$$\frac{D_2}{D_1}$$ **OPPOSITE ORDER OF FIRST RATIO**

Set the ratios equal to each other to establish an INVERSE PROPORTION STATEMENT .

$$\frac{C_1}{C_2} = \frac{D_2}{D_1}$$ **ORDER IS MAINTAINED**

Substitute the known values into the proportion.

$$\frac{C_1}{2} = \frac{12}{8}$$

Determine the CROSS PRODUCTS by using cross multiplication.

$$8C_1 = 24$$

Solve the resulting equation to determine the remaining UNKNOWN VALUE .

$$C_1 = \frac{24}{8}$$

$$\mathbf{C_1 = 3}$$

Verify the results by substituting the values back into the original inverse proportion statement.

$$\frac{C_1}{C_2} = \frac{D_2}{D_1}$$

$$\frac{3}{2} = \frac{12}{8}$$

$$\mathbf{\frac{3}{2} = \frac{3}{2}}$$

EXAMPLE.

$\mathbf{X}$ and $\mathbf{Y}$ are INVERSELY proportional.

Solve for $\mathbf{Y_2}$, if $X_1 = 9$, $X_2 = 3$ and $Y_1 = 6$.

FIRST RATIO.

$$\frac{Y_2}{Y_1}$$

SECOND RATIO.

$$\frac{X_1}{X_2}$$

INVERSE PROPORTION STATEMENT.

$$\frac{Y_2}{Y_1} = \frac{X_1}{X_2}$$

VALUE SUBSTITUTION.

$$\frac{Y_2}{6} = \frac{9}{3}$$

CROSS MULTIPLICATION.

$$3Y_2 = 54$$

EQUATION SOLVING.

$$3Y_2 = 54$$

$$Y_2 = \frac{54}{3}$$

$$\mathbf{Y_2 = 18}$$

VERIFY USING SUBSTITUTION.

$$\frac{Y_2}{Y_1} = \frac{X_1}{X_2}$$

$$\frac{18}{9} = \frac{9}{3}$$

$$\mathbf{3 = 3}$$

Just as we discovered with DIRECT proportion problems, you can begin an INVERSE PROPORTION problem in *any* one of four different ways. And, as with DIRECT PROPORTION, it makes absolutely no difference whatsoever which variable you choose to make the **FIRST RATIO** with, or the **order** in which it is written. The kicker is in the **SECOND RATIO**. **It must be made using the other variable and in the opposite order of the first ratio.** The CROSS PRODUCTS of the resulting proportional equation will be **equal** to each other. Therefore, as we experienced with direct proportions, the final result of all four possible approaches is exactly the same, as it should be, for the unknown variable can have only one value.

EXERCISE 10-4.

*Solve the following problems which involve INVERSE PROPORTION RELATIONSHIPS. Remember, **always** verify your answers by substituting all identified values back into the original proportion equation.*

1) *R and I are inversely proportional. If* $R_1 = 6$, $R_2 = 8$, *and* $I_1 = 12$, *find* I_2.

2) *D and S are inversely proportional. Find* D_2, *if* $D_1 = 4$, $S_1 = 5$ *and* $S_2 = 2$.

3) *A and B are inversely proportional. If* $A_1 = 8$, $A_2 = 3$ *and* $B_2 = 16$, *find* B_1.

WORD PROBLEMS

Now let's take a look at some practical examples of **DIRECT** and **INVERSE** PROPORTION PROBLEMS. We will pay particular attention to the assignment of letters and subscripts to the variables that are involved in the individual computations. Additionally, in these problems we *will be* including applicable units of measurement. Each problem will clearly state that the variables involved are either DIRECTLY or INVERSELY proportional. Our purpose here is not necessarily to explain the theoretical rationale behind the individual relationships; although you may already be familiar with some of the theories and their associated terms. The real objective is to provide you with experience in setting up and solving proportional problems, based on the information given. Read the given information carefully, and then follow through the example step-by-step. If at any point, the solution doesn't make sense to you . . . **STOP!** . . . go back and re-read the information provided; and start again. Not only will this type of computation make sense to you, you will probably think of many ways that you can actually put this information to good use in your daily activities.

EXAMPLE.

*The **RESISTANCE** in a piece of wire is DIRECTLY proportional to its **LENGTH**. That is, the longer the wire is, the more resistance it will have.*

*If **400 feet** of NUMBER 16 copper wire has a resistance of **3 ohms**, what is the resistance of **600 feet** of the same type and size of wire?*

*The **variables** involved here are LENGTH and RESISTANCE.*

*Let **L** represent LENGTH and **R** represent RESISTANCE.*

With the basic variables identified, you can now let $\mathbf{L_1}$ *and* $\mathbf{R_1}$ *represent the LENGTH and RESISTANCE of the FIRST piece of wire, and* $\mathbf{L_2}$ *and* $\mathbf{R_2}$ *represent the LENGTH and RESISTANCE of the SECOND piece of wire.*

Therefore, $L_1 = 400$ *feet,* $R_1 = 3$ *ohms,* $L_2 = 600$ *feet and* R_2 *is UNKNOWN.*

FIRST RATIO.

$$\frac{L_1}{L_2}$$

SECOND RATIO.

$$\frac{R_1}{R_2}$$

☜ **SAME ORDER-DIRECT PROPORTION**

PROPORTION STATEMENT.

$$\frac{L_1}{L_2} = \frac{R_1}{R_2}$$

VALUE SUBSTITUTION, INCLUDING UNITS OF MEASURE.

$$\frac{400\ \cancel{ft}}{600\ \cancel{ft}} = \frac{3\ \text{ohms}}{R_2}$$

CROSS MULTIPLICATION.

$$400R_2 = 1800\ \text{ohms}$$

EQUATION SOLVING.

$$R_2 = \frac{1800\ \text{ohms}}{400}$$

$$\mathbf{R_2 = 4.5\ ohms}$$

VERIFICATION THROUGH SUBSTITUTION.

$$\frac{L_1}{L_2} = \frac{R_1}{R_2}$$

$$\frac{400\ \cancel{ft}}{600\ \cancel{ft}} = \frac{3\ \cancel{ohms}}{4.5\ \cancel{ohms}}$$

$$\frac{2}{3} = \frac{2}{3}$$

*The 600 foot piece of #16 copper wire in question, has a resistance of **4.5** ohms.*

EXAMPLE.

*In a transformer, the **VOLTAGE** across the windings is DIRECTLY proportional to the **NUMBER OF TURNS** in the windings. If the PRIMARY WINDINGS have **100 turns** and **60 volts** present across them, and the SECONDARY WINDINGS have **150 turns**, what is the voltage present across the secondary windings?*

*Let **N** represent the NUMBER OF TURNS and **E** represent the VOLTAGE . The subscripts "p" and "s" are used to designate the PRIMARY and SECONDARY windings respectively.*

The information given in this problem can now be stated as:

N_p = 100 turns, E_p = 60 volts, N_s = 150 turns and E_s = UNKNOWN.

WRITE A DIRECT PROPORTION STATEMENT AND SOLVE FOR THE UNKNOWN VALUE.

$$\frac{N_p}{N_s} = \frac{E_p}{E_s}$$

$$\frac{100 \text{ t}\cancel{\text{ur}}\text{ns}}{150 \text{ t}\cancel{\text{ur}}\text{ns}} = \frac{60 \text{ volts}}{E_s}$$

$$100E_s = 9000 \text{ volts}$$

$$E_s = \frac{9000 \text{ volts}}{100}$$

$$\mathbf{E_s = 90 \text{ volts}}$$

VERIFY THE RESULT USING SUBSTITUTION.

$$\frac{N_p}{N_s} = \frac{E_p}{E_s}$$

$$\frac{100 \text{ t}\cancel{\text{ur}}\text{ns}}{150 \text{ t}\cancel{\text{ur}}\text{ns}} = \frac{60 \text{ v}\cancel{\text{ol}}\text{ts}}{90 \text{ v}\cancel{\text{ol}}\text{ts}}$$

$$\mathbf{\frac{2}{3} = \frac{2}{3}}$$

*The voltage across the secondary windings of this particular transformer is **90** volts.*

EXAMPLE.

*The cross-sectional area of electrical building wire is measured in circular-mils (cmil). The **RESISTANCE** of this wire is INVERSELY proportional to its **CROSS-SECTIONAL AREA**. In other words, if the cross-sectional area increases, the resistance decreases, or, if the cross-sectional area decreases, the resistance will increase.*

*A certain piece of building wire has a RESISTANCE of **8 ohms** and a CROSS-SECTIONAL AREA of **1350 cmil**. What is the RESISTANCE of another piece of wire that is the same exact length and made of the same material, but has a CROSS-SECTIONAL AREA of **450 cmil**?*

Let R_1 and A_1 represent the RESISTANCE and CROSS-SECTIONAL AREA of the FIRST wire, and R_2 and A_2 represent the RESISTANCE and CROSS-SECTIONAL AREA of the SECOND wire.

Therefore, in this example, R_1 = 8 ohms, A_1 = 1350 cmil, A_2 = 450 cmil and R_2 is the UNKNOWN variable.

Write an INVERSE proportion statement representing this relationship.

$$\frac{A_1}{A_2} = \frac{R_2}{R_1}$$

*Take note of how the **second ratio** is written . . . that's right, its in the **opposite order** of the **first ratio**, representing an **inverse** proportion.*

With the proportion statement established, we can proceed to solve for the UNKNOWN quantity.

$$\frac{1350 \text{ c}\cancel{\text{m}}\text{il}}{450 \text{ c}\cancel{\text{m}}\text{il}} = \frac{R_2}{8 \text{ ohms}}$$

$$450R_2 = 10800 \text{ ohms}$$

$$R_2 = \frac{10800 \text{ ohms}}{450}$$

$$\mathbf{R_2 = 24 \text{ ohms}}$$

Verifying through substitution.

$$\frac{A_1}{A_2} = \frac{R_2}{R_1}$$

$$\frac{1350 \text{ c}\cancel{\text{m}}\text{il}}{450 \text{ c}\cancel{\text{m}}\text{il}} = \frac{24 \text{ oh}\cancel{\text{m}}\text{s}}{8 \text{ oh}\cancel{\text{m}}\text{s}}$$

$$\mathbf{3 = 3}$$

*The resistance of the piece of wire in question is **24** ohms.*

EXAMPLE.

*In a transformer, the **CURRENTS** in the primary and secondary windings are INVERSELY proportional to the **VOLTAGES** in these windings.*

*If the PRIMARY VOLTAGE is **120 volts**, the PRIMARY CURRENT is **5 amps**, and the SECONDARY VOLTAGE is **480 volts**, determine what the current in the SECONDARY WINDING is.*

Set the variables in an INVERSE proportion statement.

$$\frac{E_p}{E_s} = \frac{I_s}{I_p}$$

Solve the proportional statement for the unknown value.

$$\frac{120 \text{ vo}\cancel{\text{l}}\text{ts}}{480 \text{ vo}\cancel{\text{l}}\text{ts}} = \frac{I_s}{5 \text{ amps}}$$

$$480I_s = 600 \text{ amps}$$

$$I_s = \frac{600 \text{ amps}}{480}$$

$$\mathbf{I_s = 1.25 \text{ amps}}$$

Use substitution to verify.

$$\frac{E_p}{E_s} = \frac{I_s}{I_p}$$

$$\frac{120 \text{ v}\cancel{\text{o}}\text{lts}}{480 \text{ v}\cancel{\text{o}}\text{lts}} = \frac{1.25 \cancel{\text{a}}\text{mps}}{5 \text{ a}\cancel{\text{m}}\text{ps}}$$

$$\mathbf{\frac{1}{4} = \frac{1}{4}}$$

The current in the secondary windings of this particular transformer is **1.25** amps.

Solving **PROPORTIONAL** problems can be a lot of fun and extremely interesting. Sometimes the relationships involved can reveal information that is very beneficial to everyday life, as well as technical computation. Now take a run at solving the problems in the following exercise . . . No doubt, you will be able to find the correct answer for *all* of them.

EXERCISE 10-5.

*Solve the following PROPORTION word problems. Be sure to show **each** step involved and verify all of your answers.*

1) *A 900 foot piece of number 18 copper wire has a resistance of 12 ohms. How long would a piece of number 18 copper wire have to be in order to have a resistance of 16 ohms? (In this case, length and resistance are directly proportional.)*

2) *A transformer has a primary voltage of 480 volts and a secondary voltage of 120 volts. If the primary windings have 700 turns, how many turns are in the secondary windings? (Voltage and number of turns are directly proportional.)*

3) *One wire has a cross-sectional area of 1250 cmil and a resistance of 7 ohms. A second piece of wire, identical except for cross-sectional area, has a resistance of 10 ohms. Determine what the cross-sectional area of this second wire is. (Cross-sectional area and resistance are inversely proportional.)*

4) *A transformer has a secondary voltage of 140 volts and a secondary current of 3.5 amps. If the primary current is 10 amps, what is the primary voltage? (In this case, the voltages and currents are inversely proportional.)*

CHAPTER 10

EXERCISE 10-1:	*1) 1 : 8 2) 8 : 1 3) 9 : 1 4) 1 : 12*
EXERCISE 10-2:	*1) 6 2) 9 3) 15 4) 12*
EXERCISE 10-3:	*1) 32 2) 49 3) 105*
EXERCISE 10-4:	*1) 9 2) 10 3) 6*
EXERCISE 10-5:	*1) 1200 FEET 2) 175 TURNS 3) 875 CMIL 4) 49 VOLTS*

Every day in the field some functions of math are being used. Most are taken for granted without much thought. Without their use however, the mechanic in the field wouldn't be able to perform the simplest task.

Where math shows its value the most however, is in Layout. Without first knowing where to build, we can't even start!

I'll take you through some of the various methods we employ in our day-to-day layout. Keep in mind most architectural drawings are dimensioned to finish so the carpenter must have the basic skills of adding /subtracting/dividing fractions and work his numbers back to actual framing lines.

Geometric Constructions

A. Triangle Use

1. To layout 90°walls- Basic 3-4-5, Pythagorean Theory $a^2+b^2=c^2$ -sheet A (Also used in our roof framing) shown later or 5-12-13
2. To layout 45°walls-1 to 1 ratio, 2 equal sides of right triangle will produce 45°
3. To layout 60°walls- 3 equal sides of triangle will produce 60°

B. Bi-sect by swing arcs

1. To produce perpendicular lines, 90°- from point on line-sheet drawing #1, #2
   - a. To produce perpendicular lines, 90°- from end of line-#3, #4
   - b. To produce perpendicular lines, 90°- from point outside of line-#5, #6
2. To layout 45°walls- #7
3. To layout 30°walls use same process to bisect 60°

C. Layout Any Degree

1. To layout any degree we scribe a radius of 57 and 5/16 inches-this we derive from 360° in a circle and we want 1 inch to represent 1°.

360 inches divide by pi (3.14)=diameter of 114.65 inches, this we divide by 2 to get the radius of 57.325 inches, now we convert to fraction of 5/16 inch. See sheet B

This brings us to our next two topics. Circles and decimal conversion, let's cover

D. Conversions

In our trade floor layout is usually in fractions but engineers use decimals for elevation marks. (Use calculator or simple math)

<u>Example:</u>

1. 97.3187 feet=97feet +, multiply .3187 times 12 (unit we want-inches) =3.8244 or 3 inches+. 8244 times 16 (unit we want. -16thof inch)=13.1904 or 13/16 of an inch-we throw away remainder and we have 97 feet 3and13/16inches. Sheet C

Drywall Layout

E. Circles

1. Pgs 10 thru 12 on drawing sheet D. This entire prelim is necessary for our many arches, vaults, Domes, etc we build.

Many times were not given the radius of segmented arches. Some of the ways we accomplish this when rise and span are given:

2. Bi-secting the chord-#8, #9

Formulas such as: $R=\frac{span^2+4(rise^2)}{8(Rise)}$ or $R=\frac{rise^2+1/2span^2}{2(rise)}$, #10, #11

Circumference is needed for aid in constructing.
C=pi (diameter)

F. Polygon Shapes

Most popular are the Hexagon (Bay windows) and Octagon (Floors).

1. Hexagon known: 6 sides all equal, 1 side =radius, 60° from center to outside points-construct from given side or circle # 12, # 13
2. Octagon known: 8 sides all equal, resides inside square, 4 sides tangent to square, size of octagon and side of square are equal. -Formulas for getting length of side of octagon from given square- #14, simple layout- #15

Armed with this information our journeyman is ready to layout this lobby. Not uncommon, dimensions are missing.

F. Drawing sheet E

1. Layout hexagon using # 16
2. Layout octagon using formula #5 on sheet # 14 or build square from known side of square per sheet # 15
3. Layout arc's per formulas on # 10, #11, calc., or # 9
4. Layout elliptical arches per Gardner method # 17 or three centered arch # 18

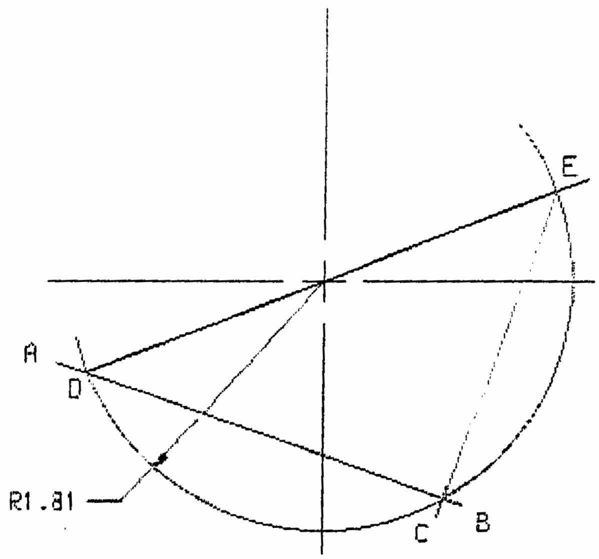
G. Conical

1. Very popular today are conical soffets. Not hard to build but here again right triangle theorems are needed to sheetrock. See Sheet F. Conicals are a part of an upside down cone-see # 19.
2. Extending lines from known points to form right triangles will give us our radius cuts for our board. i.e.: slant height=square root of h^2+r^2
   Top cut, run=4 feet, rise=5 feet, diag.=6'-4and13/16"
   Bottom cut, run=3 feet, rise =3'-9", diag.=4'-9and5/8"
3. Extent line through arc's from radius to get end cut.
4. Length of sheet is determined by Circumference length.
   C=pi (diameter) divided by 2 because of semi circle.

References: Mathematics for Carpentry from United Brotherhood of Carpenters and Joiners of America

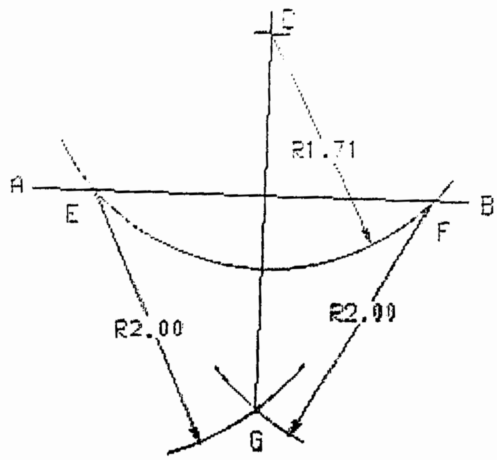
Lathers Craft Problems and Reference Book from National Lathing Industry's Joint Apprenticeship Program

- Construct a perpendicular line from a point near the end of a line -



Given Line AB and Point C, draw an arc (red) of any radius that intersects AB at points C & D; then draw a line (blue) from point D throught the center of the (red) arc until it intersects the arc again at E; then draw a line (green) at the points of intersection E to point C. CE is now perpendicular to AB.

- Construct a perpendicular line from a point to a line -



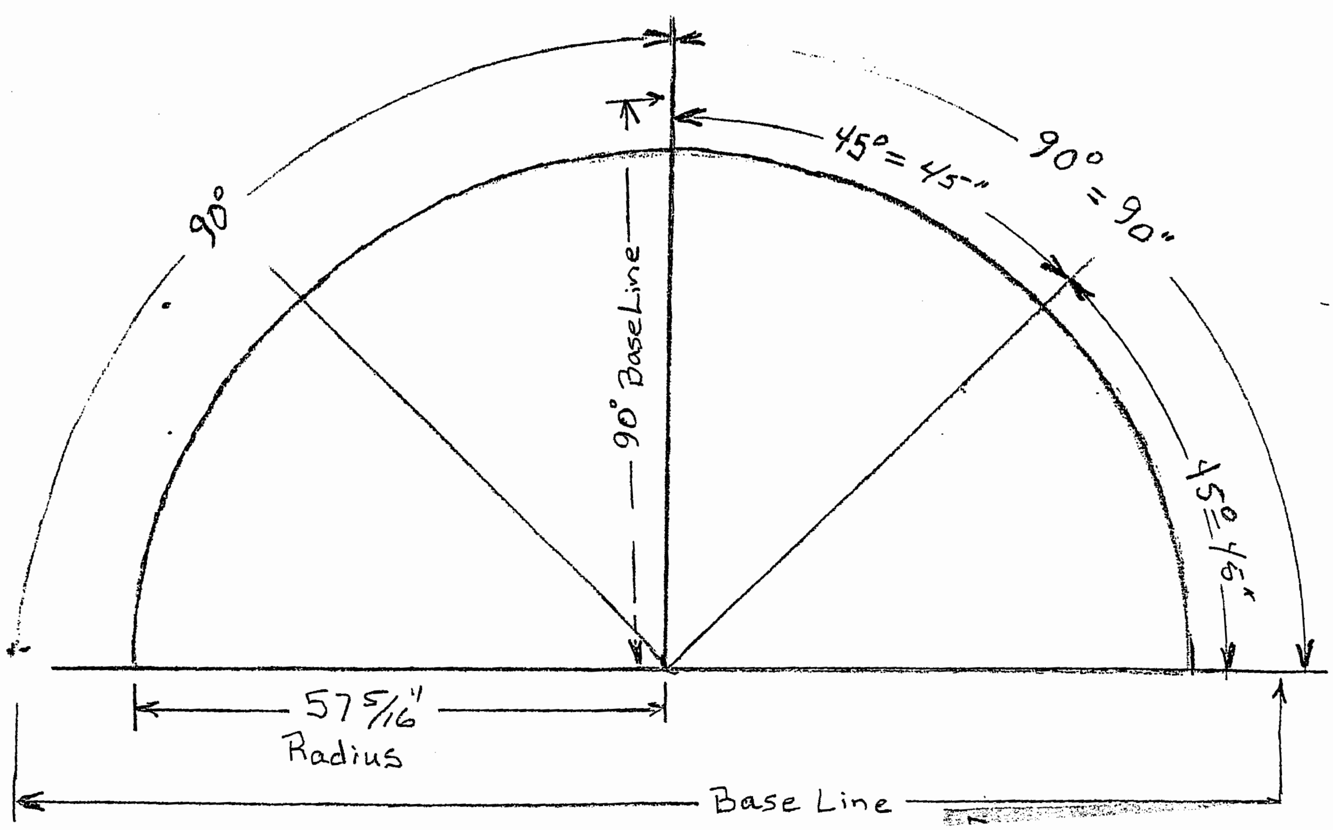
Given Line AB and Point C, draw an arc (red) of any radius that intersects AB at two points E & F; then draw two arcs (blue) EG & EF (EG=EF); and finally, draw a line (blue) at the points of intersection G to point C. CG is now perpendicular to AB.

To create a protractor during layout of walls, soffits or other items, this may be accomplished in the following way.

To create a 1 inch per degree protractor, swing an arc with a 57&5⁄16 inch radius. Then around the circumference make a mark at 1 inch intervals (i.e. 45° = 45 inches from base line).

To create a larger protractor, you may use a multiplier to make this formula work where the marks along the circumference are at 2, 3, 4 inches or greater (i.e. for 2 inch intervals the formula is: 57 5/16 inches X 2 = 114 5/8 inch radius then mark 2 inch intervals around the circumference).

1" per 1° Layout Formula



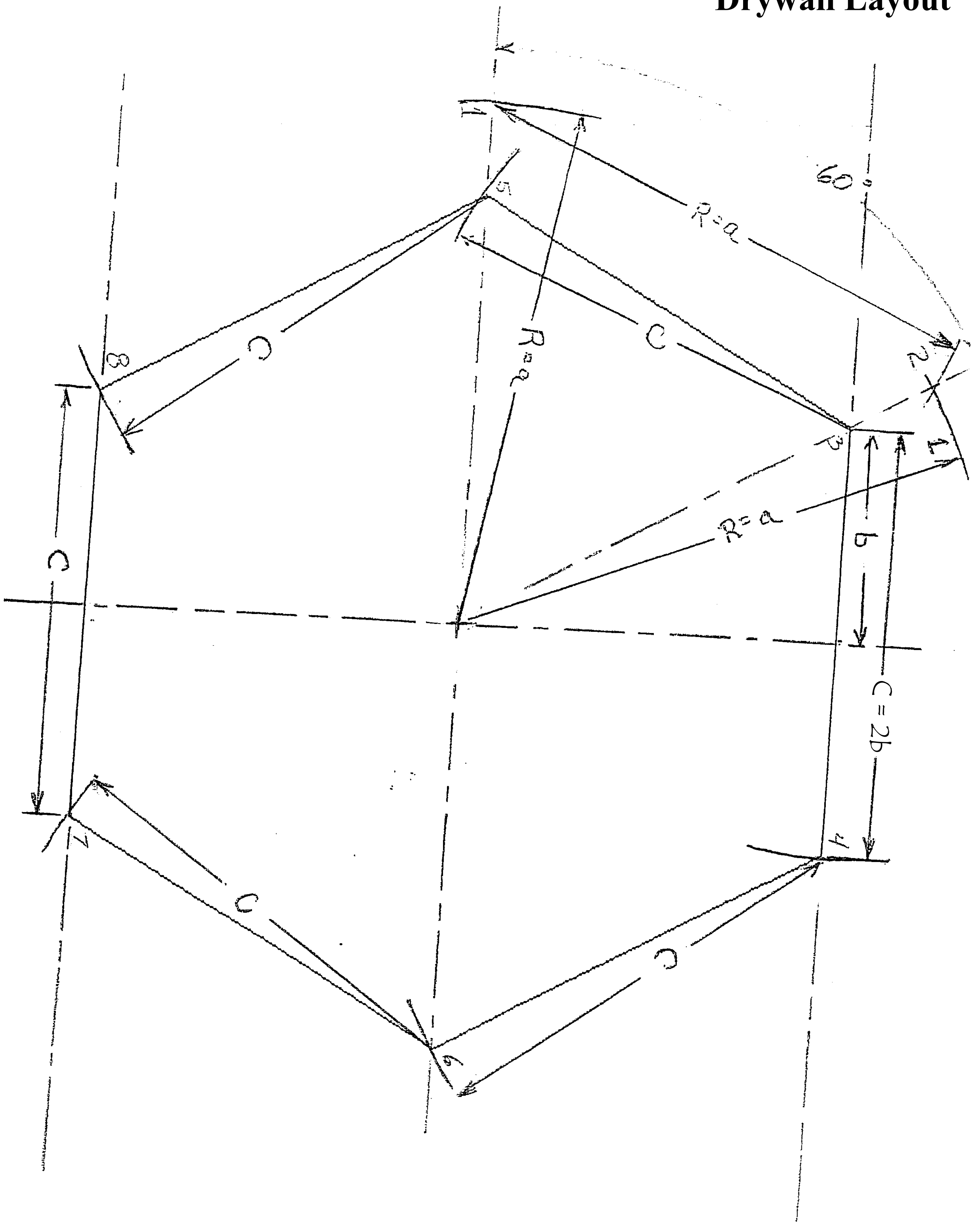
Hexagon Layout from a given Point

A hexagon is a six equal sided polygon. The following method is used to layout a hexagon when the distance between two parallel sides and a point is given.

1) Extend the given sides out in both directions.
2) Create a centerline parallel with and of the same length as the extended 2 given sides
3) Using the method of striking two equal arcs from the center point of a given side or 3/4/5 method to develop a perpendicular line to the 3 lines in step 1 & 2
4) Strike radius "a" which can be any number and gives points "1"
5) Where radius "a" intersects the centerline and continue arc ¾ the way to the perpendicular centerline
6) From the intersection of the radius on the centerline strike radius "a" to intersect with the arc from the previous step. This provides you with a point that is 60° from the centerline center point. Draw a line through the intersected arcs. Where this line intersects the given side is 30° from the perpendicular centerline and = "b".
7) Multiply "b" X 2 = "C" which is the length of all sides
8) Strike radius "C" from the intersection in step 6, point 3 to intersect centerline and the opposite end of this side line (point 4)
9) From point 4 strike radius "C" giving point 6 and from point 6 strike radius "C" giving point 7.
10) From point 3 strike radius "C" giving point 5 and from point 5 strike radius "C" giving point 8.
11) Connect points 3 / 5 / 8 / 7 / 6 / 4 and the hexagon is created

*Geometry / Math notes:
Square, pentagon, hexagon, etc. polygons sum of all angles equals equal 360°.
i.e. *Hexagon* 360°÷ 6 sides = 60°

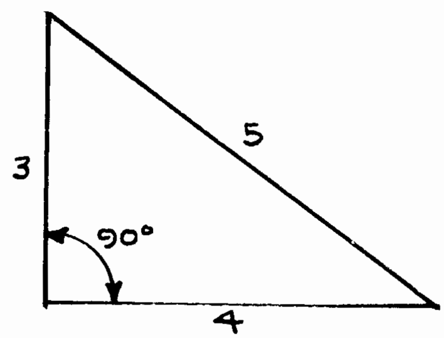
Triangles sum of all angles equals 180°
i.e. 180°÷ 3 sides = 60° (equilateral triangle)
36.87°+ 90°+ 53.13 = 180° (pythagoras theory $a^2 + b^2 = c^2$ or 3/4/5 method to provide perpendicular line to a base line).


60°
R=a
R=a
R=a
C
C
C
C
C
C
b
C = 2b
1
2
3
4
5
6
7
8

Squaring a Room

Method A: The 3-4-5 Triangle

A triangle layed off with 3,4, and 5 foot sides will have a 90° angle between the 3 and 4-foot sides.

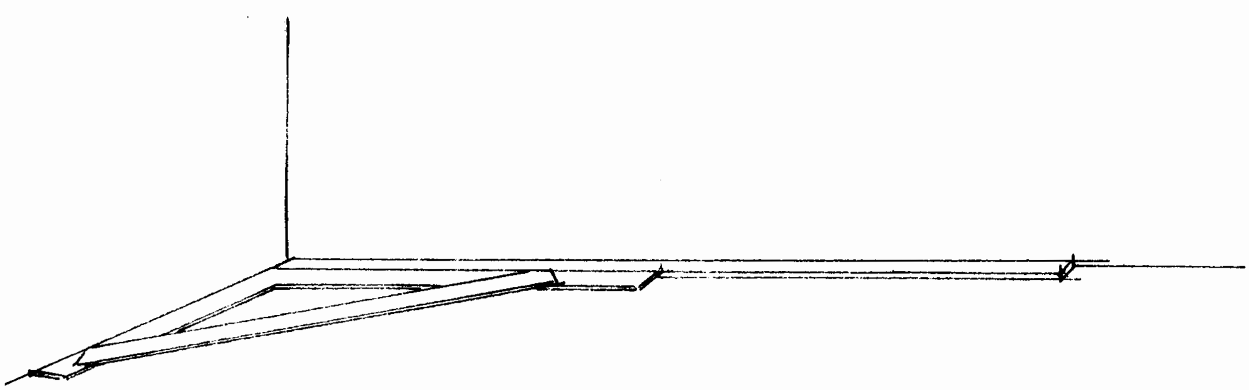


By laying this triangle off on the floor along one wall, you can see if the corner is square (90°).

Note: You can multiply the 3 - 4 - 5 by 2, making a 6 - 8 - 10 triangle or by 3 making a 9 - 12 - 15 triangle, or by 4 making a 12 - 16 - 20 triangle, and on and on. The larger triangles will be more accurate.

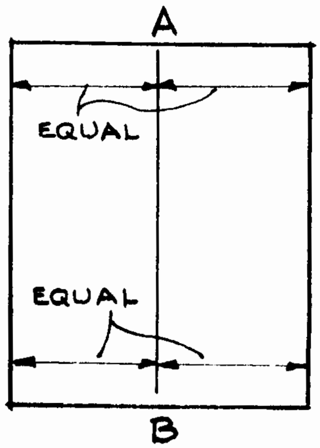
Method B: Using a large wooden square

Make a large wooden square. A long straightedge is used with the square along the edges of the room. All the corners can be checked. See below.



Method C: Bisected Center Line

(Good to use when several corners, or all corners, are out of square.)



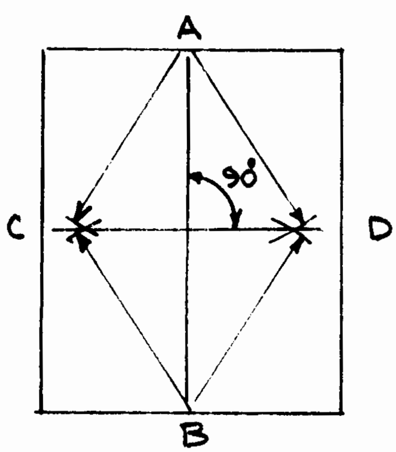
(1) Divide each end of the room into half (see points A & B at left). You must measure both ends for they may be different.

(2) Snap a line (chalk line) connecting the two points A & B.

(3) Swing an arc from both A & B, so that the arcs meet on each side of the line. See left. <u>The arcs must all be equal.</u> Call the intersection points C & D.

(4) Draw a line connecting C & D. CD is at a right angle (perpendicular) to AB.

(5) Now you can measure equal distances on both sides of AB and equal distances on both sides of CD, and layout a room with all square corners.



Painting

Fractions
- Least common denominator
- Adding and subtracting fractions
- Multiplying and dividing fractions

Temperature Conversions

Decimals
- Adding and Subtracting Decimals
- Multiplying and Dividing Decimals

Common Metric Units

Geometry Definitions
- Angles
- Polygons
- Circles

Formulas
- Perimeter
- Circumference
- Area
- Parallelograms
- Rectangles/Squares
- Circles
- Volume
- Rectangular Prisms
- Cylinders

Estimating using addition, subtraction and division

Estimating Square Footage
- Triangle
- Rectangle
- Sphere
- Piping
- Downspouts and gutters
- Chain link fences
- Picket fence
- Walls
- Stacks
- Irregular Shapes
    - Corrugated metals
    - Roof deck
    - Siding

Tacoma Community College Performance Task (NOT STATE)

Created/Revised 10/07/2004

Performance Task Title: *"Changing Spaces" in Your Home*

General Purpose of Performance Tasks: To determine if the learner has made progress in or completed ABE Math Level IV.

Content Area and Level: ABE Math Level IV

Overall Conditions for Use:

- ☑ Insofar as possible, the task is performed in a distraction-free environment.
- ☑ Each performance must be an observed event completed within one class session.

I. Provide a brief description of the performance task (including the specific purpose, and specific observable outcome or product that will be scored):

Students will show they can do calculations related to area, percents, costs and conversions between units of measurement in retiling and carpeting a home.

II. List the competencies (State & CASAS) that are incorporated into the task (For example: S3.2 - make a personal excuse or request):

STATE COMPETENCIES

4.2.0 Communicate an understanding of the percent concept and how percents relate to such consumer issues as purchasing, budgeting, and managing finances

4.3.0 Demonstrate when and how to use fractions, decimals, and percents interchangeably

4.4.0 Identify and solve for the whole, part, and percent to make informed financial decisions

4.5.0 Use formulas for perimeter, area, and volume appropriately to select and evaluate goods and services

4.5.1 Convert among U.S. standard measurement

CASAS COMPETENCIES

1.1.4 Select, compute, or interpret appropriate standard measurement for length, width, perimeter, area, volume, height, or weight
6.0.1 Identify and classify numeric symbols
6.0.3 Identify information needed to solve a given problem
6.0.4 Determine appropriate operation to apply to a given problem
6.1.1 Add whole numbers
6.1.3 Multiply whole numbers
6.1.5 Perform multiple operations using whole numbers
6.2.1 Add decimal fractions
6.2.3 Multiply decimal fractions
6.2.5 Perform multiple operations using decimal fractions
6.4.2 Apply a percent in a context not involving money
6.4.3 Calculate percents
6.4.4 Convert percents to common, mixed, or decimal fractions
6.4.5 Use rate to compute increase or decrease
6.5.2 Recognize and apply simple geographic formulas
6.6.2 Recognize, use, and measure linear dimensions, geometric shapes, or angles
6.6.3 Measure area and volume of geometric shapes
6.6.5 Interpret diagrams, illustrations, and scale drawings
6.6.7 Solve measurement problems in stipulated situations
6.6.8 Interpret mechanical concepts or special relationships
7.2.2 Analyze a situation, statement, or process, identifying component elements and causal and part/whole relationships
8.2.6 Recognize and/or demonstrate general household repair and maintenance

III. Explain what knowledge and skills are required for successful performance of this task.

- Knowledge of all basic mathematics symbols
- Ability to perform with accuracy all four basic mathematics operations using whole numbers, decimals, and percents
- Can interpret and apply geometric formulas
- Can understand when to use area, percents, and conversion formulas to solve applied math problems
- Identify the main issues in complex situations
- Evaluate the costs and benefits of choices and actions
- Effective communication
- Critical thinking in considering qualitative and quantitative approaches

IV. Describe the learning activities that you and your learners have been engaged in to develop the skills necessary to accomplish this task:

- Rubrics and competencies
- Practice learning and using the four basic mathematics operations
- Solve problems to demonstrate this capacity in real-life situations
- Exercises in calculating area, percent, and conversions of U. S. standard units of measure

V. Context: Explain how the task is relevant to the learners' lives and learning goals, and whether the skills needed for the task are applicable to the learners' real life Work, Family and/or Community roles.

This task is relevant to the learners' lives and learning goals since they can determine and practice the necessary steps in making improvements to their homes while utilizing math concepts. These skills are applicable to the learners' real-life Work Family and/or Community roles, since they can be used in working in the building trades or in working for companies that retail products for home improvement projects, can involve family members in selecting materials and comparing costs for making changes in their home environments and can also be used for community improvement projects.

VI. Instructions for Teacher:

- Be certain that the students understand that they will be assessed and understand the rubric criteria
- Use clear and simple instructions
- Have all materials available
- Use scoring sheets to give students feedback
- Inform students when they will be given the results
- Remind students to show their work

VII. Materials and Resources:

- Paper, pencils, erasers
- Scratch paper
- Handouts with clear instructions
- Rubrics with self-assessment
- Formula for the area of rectangles, and U. S. standard measurement conversions

VIII. Conditions for Task Use in Specific Subject Level:

- List of formulas available
- Calculators may be used (added site condition)
- Tools acceptable where appropriate
- Untimed, but within one class period or tutor session
- Assistance allowable but determines stage
- Scratch paper allowed, show work

Any type of paper acceptable

IX. Instructions for Learner:	
The instructions are appropriate for the level of the learner • This performance task will be used to determine the progress within your level • It will be scored according to the rubric we have discussed in class • Read the instructions carefully • You will have one class session to complete this task • You must show your work in the space provided on the test sheets	
X. Explain how and when the expected product and criteria for evaluation will be made clear to learners:	**XI. Describe how a learner reflection / self-assessment activity will be incorporated into the task and/or learning activities:**
The holistic rubric and the competencies on which they are based are a regular part of the instructional cycle. The students know what their level is, they understand that the learning activities are geared to the competencies, and the performance task is a culminating event used to determine progress.	During the rubrics/competencies review at the beginning of the learning cycle, time is given for a discussion on the impact of this process on the learner. There will be an opportunity for both group and individualized debriefing.

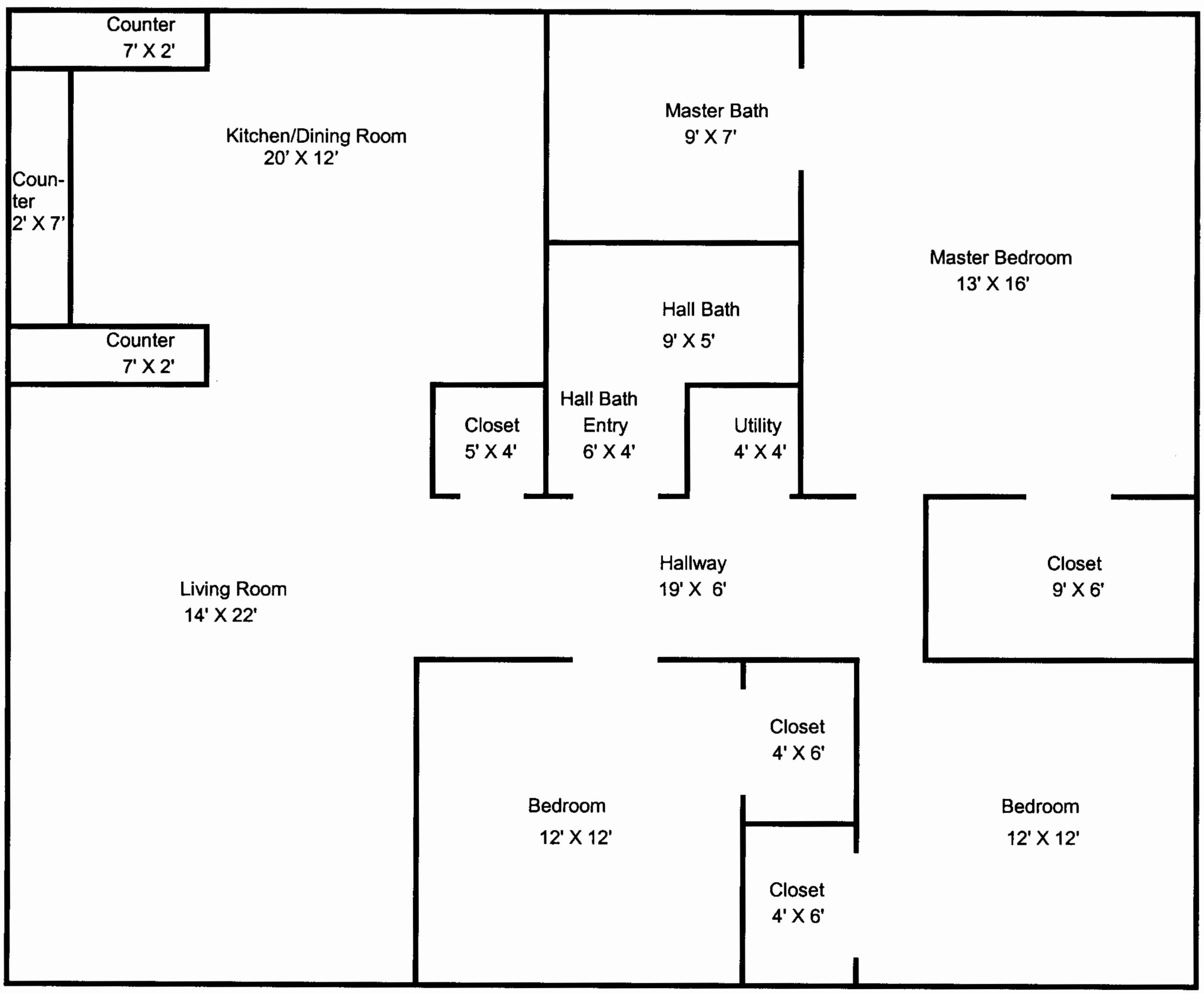
Carpet Laying and Tile Setting

Name: ______________________________ **Date:** ____________________

ABE Level IV Math Assessment Task
"Changing Spaces" in Your Home

This exercise will assess your ability to solve real-life math problems using basic mathematics operations for calculations of area, percents and conversions between U. S. standard units of measurement. You must show your work for each problem in the space provided.

Scenario: You have just purchased a home that needs interior remodeling. Your family decides to install new tile and carpet on the floors. A sketch of the floor plan is shown below. All measurements are in feet and represent the measurements rounded up to the nearest foot.



Carpet Laying and Tile Setting

Name: ______________________________ **Date:** __________________

ABE Level IV Math Assessment Task
"Changing Spaces" in Your Home

1. What is the square footage of your home?

2. How many square feet of floor space are in each of the following areas?

   a. Kitchen/dining room

   b. Living room

   c. Master bath and hall bath (including hall bath entry) combined

   d. Hallway, hall closet and utility room combined

3. You have decided to install floor tiles in the kitchen/dining room, living room, both baths, hall bath entry, hallway, hall closet and utility room. The flooring company salesperson recommends that you increase your square footage requirement by 5% to compensate for waste. How many square feet of tile are needed?

4. Tiles are sold in boxes of twelve one-foot square tiles and partial boxes are not sold. The tile that you selected costs $5.75 per square foot, installed, based on the total number of tiles purchased. The sales tax rate is 8.25%.

   a. How many boxes of tile must you purchase?

   b. How much will it cost to install the tile, including sales tax?

Carpet Laying and Tile Setting

Name: ______________________________ **Date:** ________________

ABE Level IV Math Assessment Task
"Changing Spaces" in Your Home

5. How many square feet of floor is in all the bedrooms and bedroom closets, combined?

6. You plan to carpet the bedrooms and closets. The carpet costs $24.95 per square *yard,* installed, and the sales tax rate is 8.25%. The carpet is sold in rolls that are 12 feet wide, so a 3 foot length equals 4 square yards. The salesperson recommends that you purchase a 45 foot length of 12 foot wide carpet.

    a. The 45 foot length of carpet is how many square *yards* of carpet? (Hint: There are 9 square feet per square yard.)

    b. How much will it cost to install the carpet, including sales tax?

7. What is the total cost (including tax) for the new tile and carpeting in your home?

ROOF FRAMING

The ability to calculate, layout, and frame roofs is one of the defining skills of a carpenter. One needs good visualization skills, because roof systems are three-dimensional, and a good understanding of geometry because most roofs are based on the right triangle.

Once the walls are erected and plumbed, the carpenter must begin collecting field measurements and roof framing information off of the blueprints such as roof rise and run. Next, calculations will be carried out to determine the theoretical measurements necessary to layout the framing members, and finally the carpenter must layout, cut and install the rafters.

Our mission is to learn the vocabulary of roof framing, collect information and calculate the measurements necessary to layout roof members. Rafter layout requires more skills you will learn later.

ROOFS

Roofs come in many different shapes and styles but by in large they are nothing more than right triangles. *Figure 10-8* illustrates the simplest roof form, the **shed roof.** *Figure 10-9* illustrates the right triangle hiding under the roof.

Depending on the roof, the right triangle can have different proportions. Those proportions are defined by the **unit triangle,** pictured in *Figure 10-8.* The unit triangle defines the rise and run of the roof in the simplest form. For shed and gable roofs the run will always be 12″ and the rise will vary. The unit triangle in figure 10-8 has a run of 12 ″ and a rise of 6″. In terms of geometry we would say that the triangles base is 12″ and its altitude is 6″.

The most important thing to remember is that the geometry of the unit triangle is exactly the same as the geometry of the roof you are calculating; the dimensions are different but proportionate.

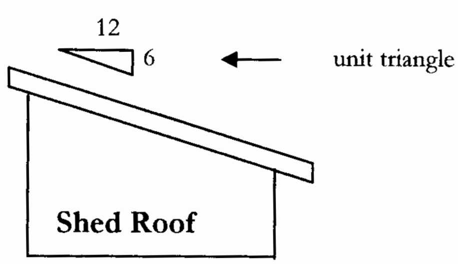


FIGURE 10-8

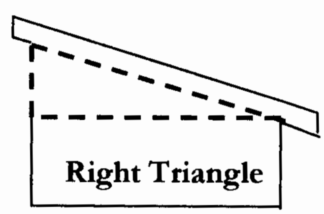


FIGURE 10-9

The gable roof (*Figure 10-10*) is made up of two right triangles.

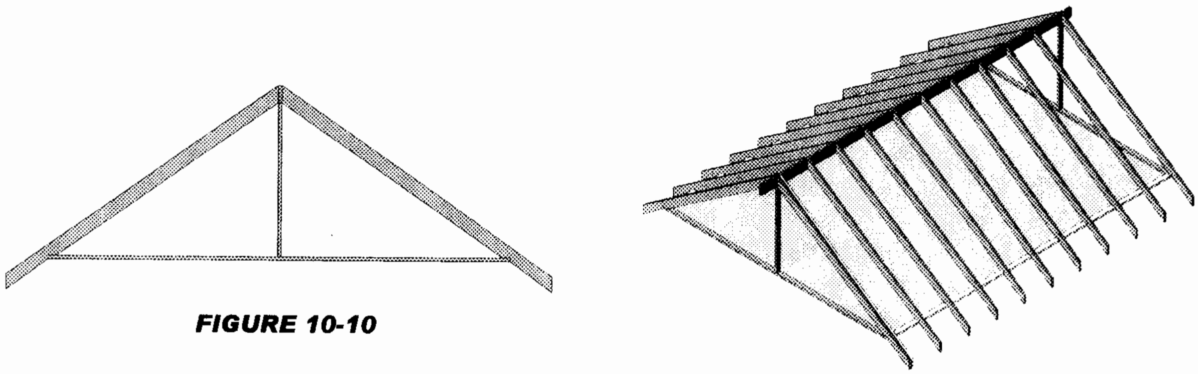


FIGURE 10-10

NOTE: ***Other roof styles will be discussed later in this chapter.***

ROOF TERMINOLOGY AS SEEN IN *FIGURE 10-11*

It is important for you to know and understand the basic terminology used in roof framing.

Span The total width of the structure
Total Run One half the span (base of the roof triangle)
Total Rise The height or altitude of the roof triangle
Unit Run The unit of measurement given in inches
Unit Rise The amount of rise per foot of run
Slope* The incline of a roof expressed as a ratio of rise to run. i.e. 4/12
Pitch* The slope of a roof expressed as a ratio of rise to span
Line length The hypotenuse of the roof triangle

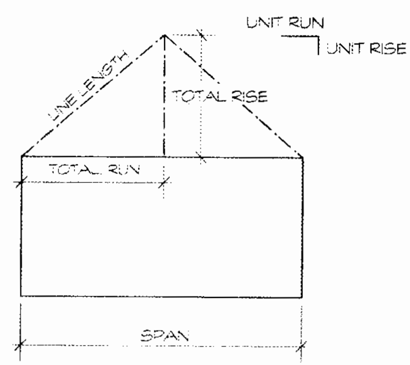


FIGURE 10-11

NOTE:** ***The words slope and pitch have different meanings as noted above, however you will commonly hear them used interchangeably. We will not speak in terms of roof pitch in this book because it is rarely used. Later we will talk about slope angle and how it may be calculated.

13. Write in the roof properties listed below in the appropriate locations on *Figure 10-12.*

Properties
Span = 24′-0″
Run = 12′-0″
Rise = 6′ 0″
unit run = 12″
unit rise = 6″
*Slope = 6 /12**
*Pitch = 6/24 Or .25**
Line Length = 13′- 5″
* do not include in diagram

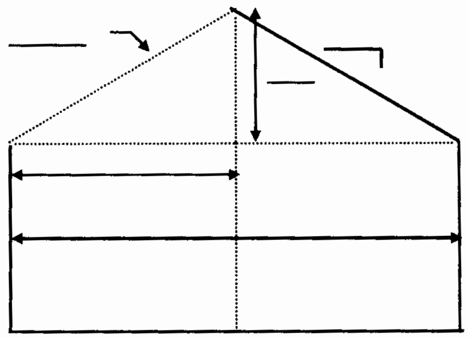


FIGURE 10-12

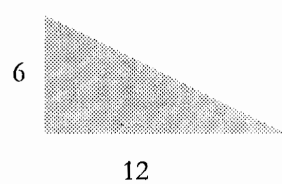
CALCULATING ROOF PROPERTIES

Typically, roof span and the unit triangle can be found on your plans, but its line length and rise which must be calculated.

Armed with unit rise, unit run and total run or span we have all the information needed to calculate total rise, and rafter length for any basic roof. **The unit triangle is proportionate to the actual roof triangle**. By setting up ratios and factors (constants) as we did earlier, we can easily calculate the lengths of all our roof members. Let's use the 6/12 slope triangle to show how easy it is to calculate total rise and line length.

Note: *The term constant will be used in place of the term factor in this chapter; because once the factor is calculated for each roof slope it never changes making it a constant. See the definitions in the glossary.*

TOTAL RISE CALCULATION – In the case of the 6/12 roof, it is easy to see that the unit rise is one-half the unit run. *Figure 10-13* shows how the constant is calculated. The constant can now be multiplied times any run to obtain a proportional rise. For example, a total run of 20′-0″ has a total rise of 10′-0″. **20 x .5 = 10 feet.**



Unit Triangle

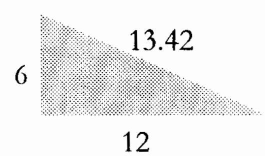
6/12 = .5
.5 is the rise constant for any roof with a slope of 6/12

FIGURE 10-13

REMEMBER: ***The ratio is set up with the value you want to know as the numerator. The ratio above is 6/12 because we want to know the total rise. If you wanted to solve for the total run you could set up the ratio as 12/6. 12/6 = 2 Therefore a building with a total rise of 10 feet will have a total run of 20 feet. 10 x 2 = 2***

LINE LENGTH

As defined earlier, line length is the hypotenuse of a right triangle. Again, the hypotenuse of the unit triangle is proportionate to that of the roof triangle. So, just as we did before we can set up a ratio with the unit hypotenuse as the numerator and the unit run as the denominator as in *Figure 10-14*. The resulting constant, when multiplied times the total run, yields the line length of the common rafter. All we have to do is calculate the hypotenuse using the Pythagorean Theorem and divide by 12.



13.42/12 = 1.12
1.12 is the line length constant for any roof with a slope of 6/12

FIGURE 10-14

Homework:

Calculate the rise and line length constants for roof slopes 2/12 through 14/12 and add to your reference manual.

TRY THESE:

Where appropriate express answers in feet and inches

14. The span of a structure is 32'-0" and the slope is 4/12. Fill in the blanks below.

Rise constant________ Total run ____________ Line length ___________

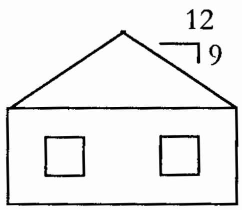
Line length constant _____________ Total rise _____________

15. The total run of a building is 17'-5" and the slope is 8/12. Fill in the blanks below.

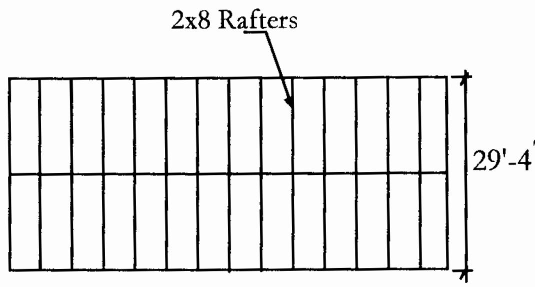
Span ________ Rise constant ________ Line length _________

Line length constant _____________ Total rise _______________

16. Fill in the blanks based on the information given in illustration.



Elevation



Plan View

Rise constant ________ Total run ___________ Total rise ___________

Line length constant ____________ Line length __________

A CLOSER LOOK AT RAFTER CALCULATIONS

Until now we have been looking at simple wire frame models of framed roofs but in reality rafters have thickness and height so let's take a closer look. *Figure 10-15* introduces us to some new terminology.

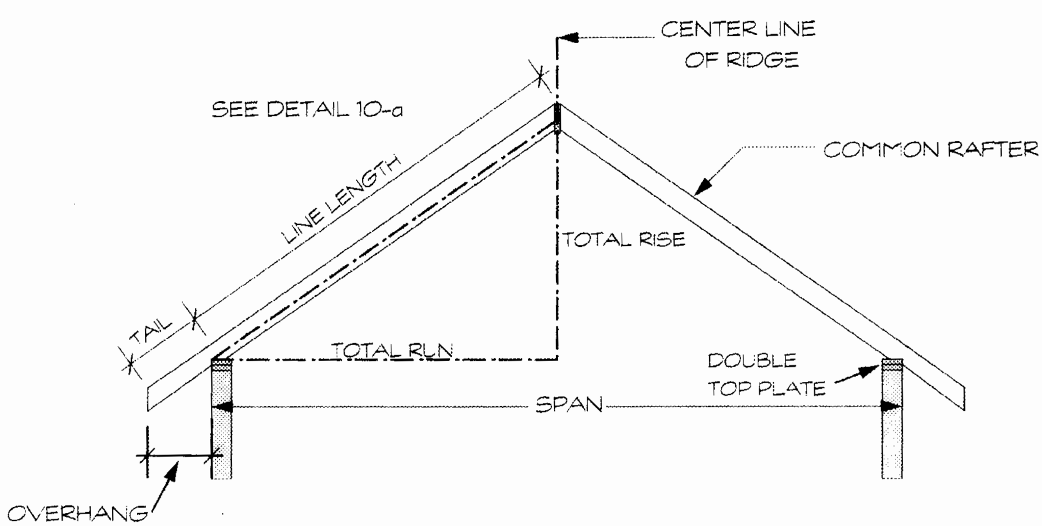


FIGURE10-15

In the previous examples you solved the theoretical line length and theoretical total rise. It is important to note where the theoretical triangle is in relation to the actual rafter. Notice in *Figure 10-15* the theoretical line length or layout line is not measured at the top or bottom edge of the rafter. ***Detail 10-a*** on page 103 clearly shows the point to which total rise is measured at the ridge and the top plate of the wall.

If you look carefully at *Figure 10-15* you can see the overhang extends the rafter beyond the wall plate. Note also that the rafter must be shortened at the ridge because the ridge board has thickness.

More Roof Terminology

- **The Rafter Tail or Projection** – The line length of the rafter does not include the tail. The length of the tail is a separate calculation, based on the roof overhang . *See Figure 10-1*5

- **Overhang** – The overhang is the horizontal measurement from the wall to the end of the rafter tail.. If the plan shows the overhang at 24", that means 24" horizontally from the vertical plane of the wall to the vertical plane of the rafter tail. *See Figure 10-15*

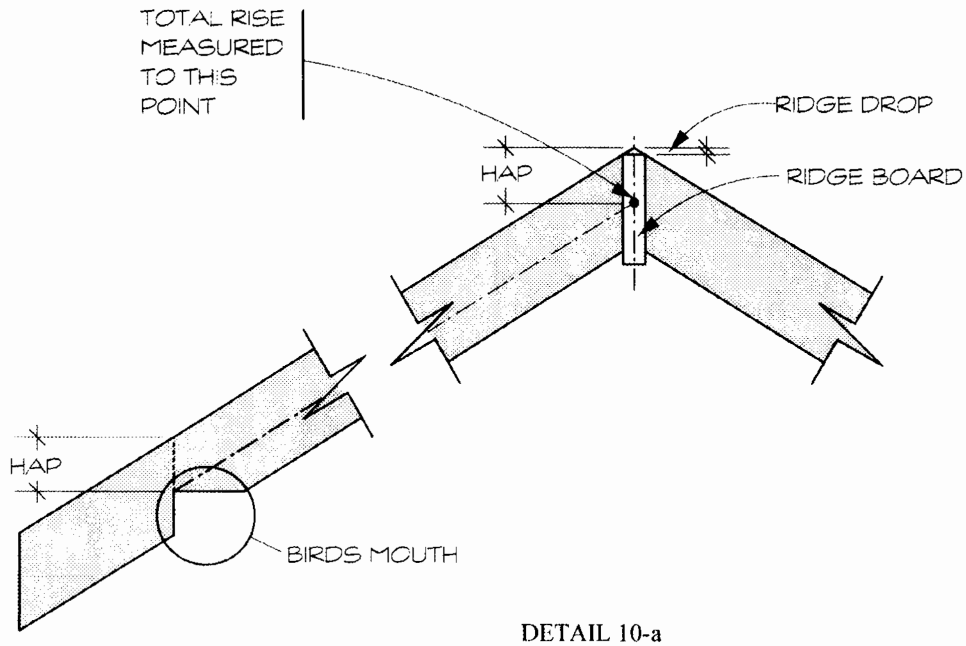
- **Bird's Mouth** – This is a notch cut into a rafter to provide a bearing surface where the rafter intersects the top plate of the wall. The bird's mouth notch is comprised of two cuts in the rafter: A **seat cut,** which is the horizontal cut where the rafter bears on the top wall plate, and the **plumb cut,** which as the name implies, is the vertical cut of the bird's mouth. ***See Detail 10-a***

- **Height at Plate (HAP)** The distance measured vertically from the intersection of the seat cut and plumb cut, to the top edge of the rafter. ***See Detail 10-a.***

  The "HAP" measurement is a variable determined by the carpenter on the job. Generally speaking it should be no less than 2". Notice that the "HAP" measurement is the same at the ridge end of the rafter.

- **Ridge** – The horizontal framing member that rafters are aligned against to resist their downward force. The minimum thickness allowed by the *International Structural Code for One and Two Family Dwellings* is 1" nominal, however 2x members are more typically used. Notice that the ridge has been lowered so that its top edges align with the top edges of the rafters. Beveling the ridge to match the angle of the rafter would eliminate its lowering, however this is rarely done because of the labor-intensive nature of such a procedure.

NOTE: ***Another building code issue worth mentioning is that rafters must have full bearing against the ridge board. Generally, the rafter is a size smaller than the ridge board on which it bears.***



DETAIL 10-a

Example 10-D: You must construct a roof with a slope of 8/12, a span of 25'-0", and an overhang of 24". The plan calls for 2x8 rafters a 2x10 ridge board and a 4" HAP.

Here is what you want to know:

- To what length will the rafter be cut, including the tail?
- What is the total rise?
- How far will the ridge be dropped?
- You must cut a 2x4 brace, running from the top plate to the bottom of the ridge, to temporarily support the ridge until the rafters are in place. To what length will you cut the 2x4 brace?

Step 1: Make a sketch of the roof showing what you know. This is a good idea until you become very comfortable with these calculations.

Sketch Here

Step 2: Calculate the rise and line length constants

Rise Constant = 8/12 = .6667

Line Length Constant = $\dfrac{\sqrt{8^2 + 12^2}}{12} = 1.2019$

Step 3: Calculate the total run.

Formula: Span + 2 = Total run

25' ÷ 2 = 12.5' or 12' 6"

Total run = **12'- 6"**

Step 4: Calculate the total rise

Rise Constant × Total Run

.6667 ×12.5′ = 8.33′

Total Rise = **8'- 4"**

Step 5: Calculate the line length *(remember the line length constant is the hypotenuse divided by 12)*

Line Length Constant × Total run

12.5′ × 1.2019 = 15.0238

Rafter line length = **15′- 0 1/4"**

Step 6: Calculate the line length of the rafter tail.

Line Length Constant × Overhang

1.2019 × 2′ = 2.4038

Line length of rafter tail = **2'- 4 7/8"**

Step 7: Total Length of Rafter

15'- 0 1/4"
\+ 2' - 4 7/8"
17' -5 1/8"

Total length of rafter = 17'- 5 1/8"

Step 8: Calculate ridge drop. If there were no ridge board, the rafters would meet at the centerline, but because the ridge board has thickness the rafters are backed off horizontally, one-half the thickness of the ridge. See *Figure 10-16*

You can see in *Figure 10-16* that as a result of backing the rafters away from the centerline, the top edge of the rafter drops. You could cut a bevel on the top edges of the ridge to continue the rafter angle to the centerline, but that would be quite labor intensive.

So, the question is - **How much do we drop the ridge so that its top edge meets the top edge of the rafter?** Looking at *Figure 10-16* you can see that the amount of drop needed is the altitude of the small black triangle labeled Triangle A.

As you might have already figured out this little triangle, tiny as it is, is proportional to the unit triangle. We can calculate the rise of triangle A by multiplying the rise constant times its run.
The total run in this case is half the thickness of the ridge or $^3/_4$".

Rise Constant × Total Run

.6667 × .75 = .5

Ridge Drop = $^1/_2$"

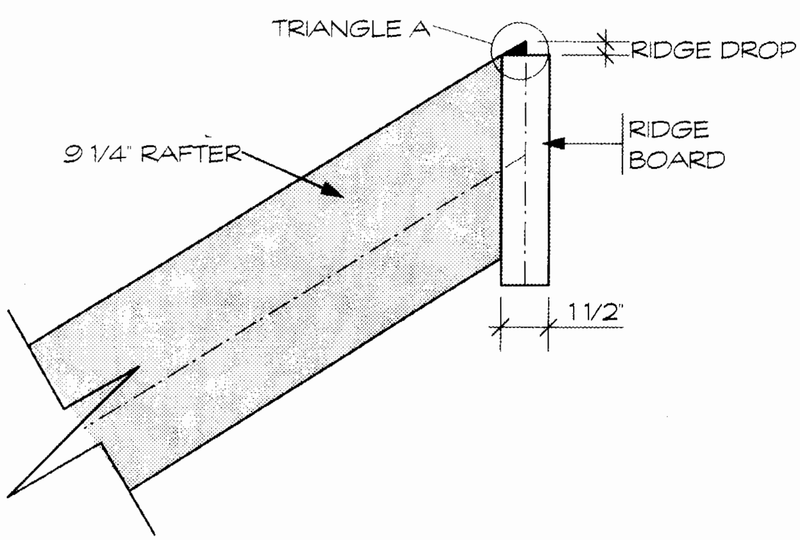
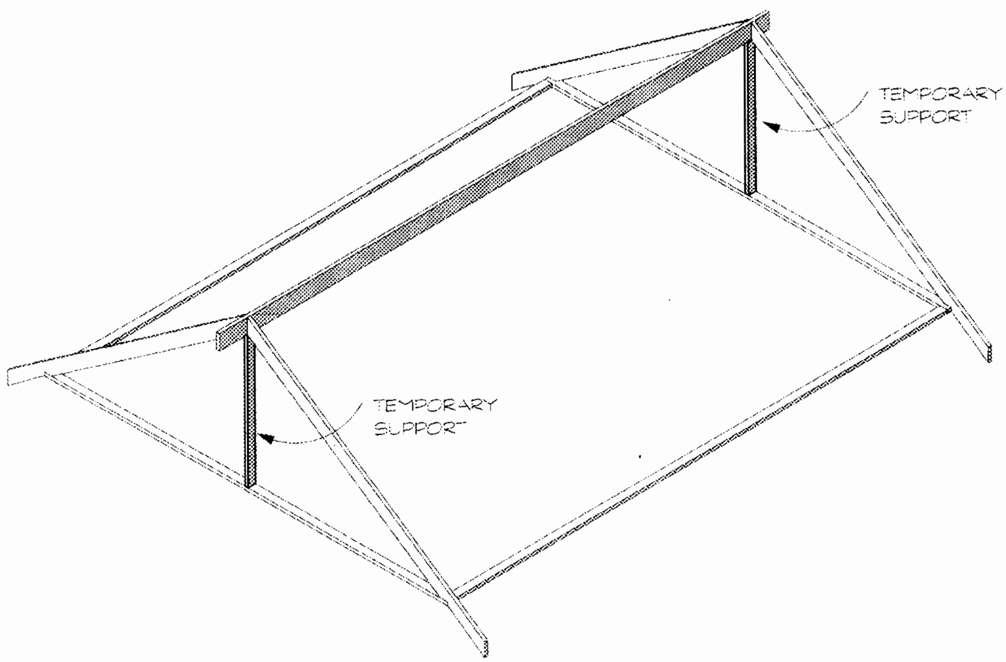


***FIGURE* 10-16**

The first element to be erected when framing a roof is the ridge board because without it, the rafters have nothing to rest upon. The ridge is usually supported at each end by temporary vertical support (i.e. 2x4) that is supported by the wall beneath. ***See the illustration below.*** In Step 9 we will calculate the length of the temporary support.



Step 9: Calculate the length of the temporarily ridge support.
Remember the support extends from the top plate of the wall to the bottom of the ridge board.
Also remember that the total rise extends from the top plate to the point where the layout line intersects with the centerline of the ridge. To solve for the support length we must add the height at plate (HAP) measurement to the total rise, subtract the ridge drop and finally subtract the height of the ridge board.

We already have all the information we need:

Total rise	⟶	8′- 4″
HAP	⟶	+ 4″
Total Rise + HAP	⟶	8′ -8″
Ridge Drop	⟶	- 0 $^{1}/_{2}$″
		8′ -7 $^{1}/_{2}$″
Ridge Height	⟶	- 9 $^{1}/_{4}$″
Length of 2x4 support	=	**7′- 10 $^{1}/_{4}$″**

NOTE: ***Working several of these problems will bring all of this into focus. There is much more involved in actually laying out and cutting common rafters. The purpose of this chapter is to focusing on the theory behind roof framing. An understanding of this theory will make rafter layout much easier and fun.***

Are you ready to try some on your own? Use the previous example and your roof constants to guide you through the next three problems

17. Situation: **Fill in the blanks:**

Situation	Fill in the blanks	
• Gable roof		
• Span = 28′-0″	Total rise	________
• Slope = 4/12	Line Length	________
• HAP = 5″	Tail line length	________
• Overhang = 18″	Total rafter length	________
• Rafters = 2x10 (1 ½ x9 ¼)	Ridge drop	________
• Ridge = 2x12 (1 ½ x 11 1/4"	Support length	________

- The ridge is to have a temp. brace supported at the top plate.

18. Situation: **Fill in the blanks:**

Situation	Fill in the blanks	
• Shed roof		
• Total run = 15′-9″	Total rise	________
• Slope = 7/12	Line length	________
• HAP = 3 1/2″	Tail line length	________
• Overhang = 16″	Total rafter length	________
• Rafters = 2X8 (1 ½ x7 ¼)		
• Ridge = 2X10 (1 ½ x 9 ¼)		
• No temp. brace necessary		

19. Situation: **Fill in the blanks:**

Situation	Fill in the blanks	
• Gable roof		
• Span = 22′-8 5/8″	Total rise	________
• Slope = 6/12	Line Length	________
• HAP = 3″	Tail line length	________
• Overhang = 18″	Total rafter length	________
• Rafters = 2x8 (1 ½ x7 ¼)	Ridge drop	________
• Ridge = 2x10 (1 ½ x 9 ¼)	Support length	________

- The ridge is to have a temporary brace supported at the top plate.

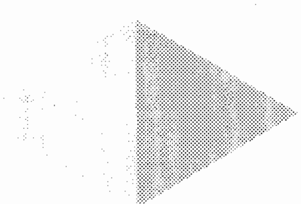
20. Situation: *Hint – find a proportion to solve total run* **Fill in the blanks:**

• Gable roof		
• Span unknown	Total run	__________
• Total run unknown	Line Length	__________
• Slope = 10/12	Tail line length	__________
• Total rise = 9′-5″	Total rafter length	__________
• HAP = 4″	Ridge drop	__________
• Overhang = 28″	Support length	__________

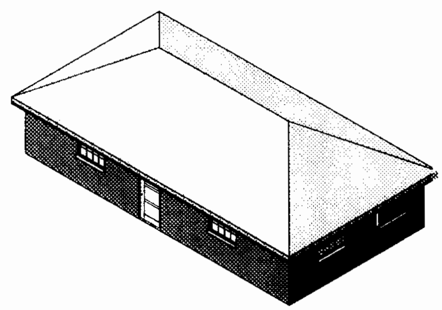
- Rafters = 2x12 (1 ½ x 11 ¼)
- Ridge = 2x14 (1 ½ x 13¼)
- The ridge is to have a temp. brace supported at the top plate

THE HIP ROOF

A hip roof is a roof shape with four sloping sides. The most basic form of hip roof is a four-sided pyramid illustrated in *Figure 10-17.* Typically hip roofs look more like the illustration in *Figure 10-18.*



Pyramid
***FIGURE* 10-17**



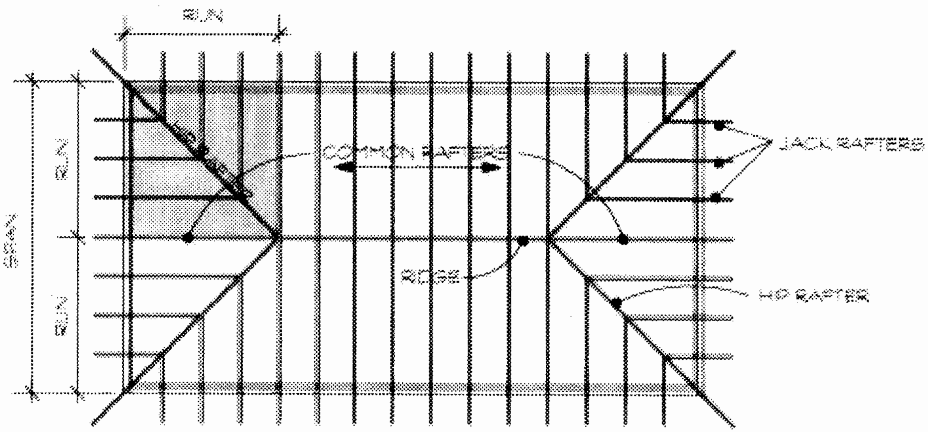
Hip Roof
***FIGURE* 10-18**

The rafter layout of a hip roof is illustrated in plan view in *Figure 10-19.* Notice that each end of the roof has been divided into two squares with sides equaling the total run dimension (*see the gray square in figure 10-19 for clarification*). A hip rafter cuts a 45° diagonal across each square. Another way of saying this is that the hip rafter is the diagonal of a square, having sides equal to the total run.

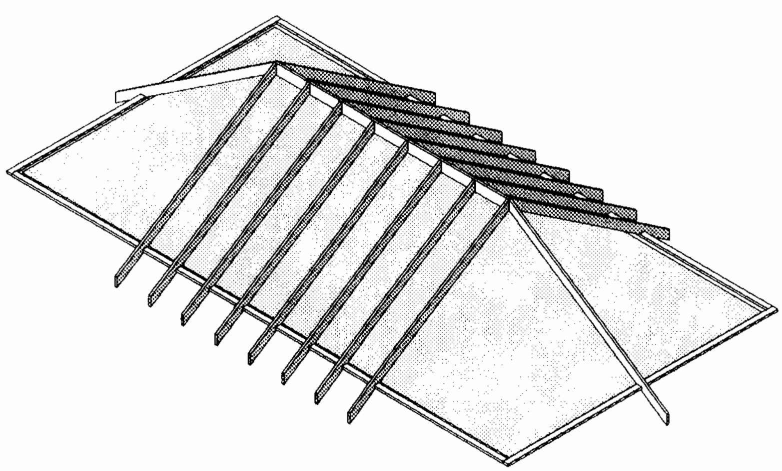
Let's take the roof apart and look at the individual components. The isometric view in *Figure 10-20* shows the common rafters and ridge. Notice there are 8 common rafters on each side of the ridge and two parallel to the ridge.

Figure 10-21 shows the hip rafters running diagonally across the square and rising to the ridge board. We will come back to this later. *Figure 10-22* shows the hip rafters and jack rafters together. Notice that jack rafters are simply shortened common rafters, because of their intersection with the hip.

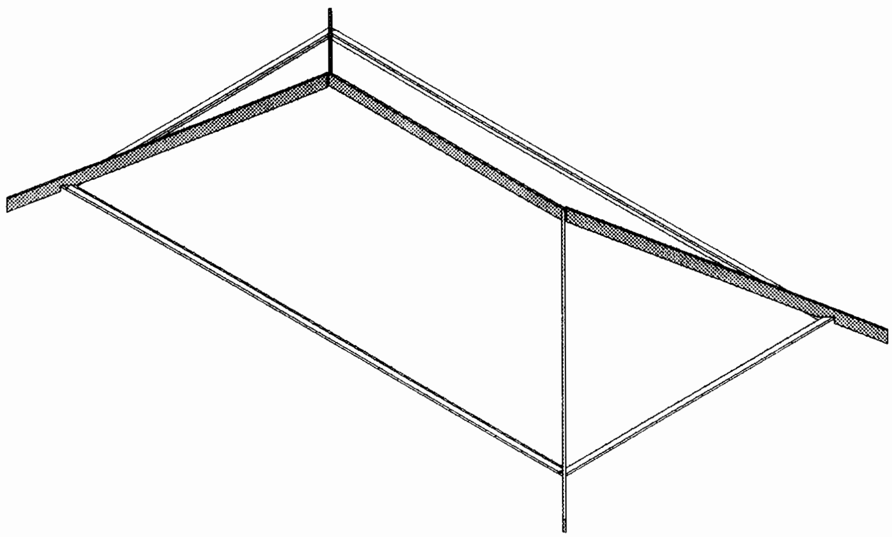
Finally in *Figure 10-23* common rafters have been added to complete the roof frame. Again notice that the jacks are simply common rafters that, because of their intersection with the diagonal hip rafter, get progressively shorter in length.



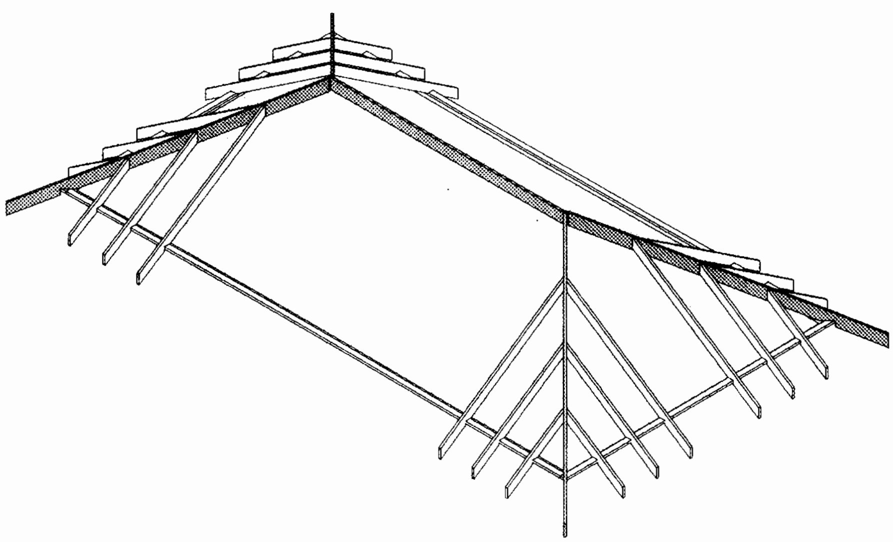
Plan View of Hip Roof
FIGURE 10-19



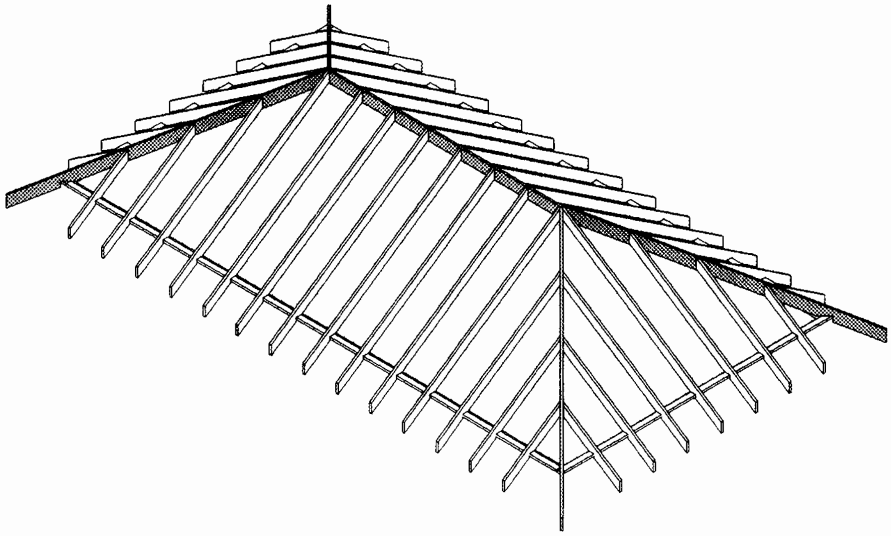
Ridge w/ Common Rafters
FIGURE 10-20



Ridge w/ Hip Rafters
***FIGURE* 10-21**



Ridge w/ Hip Rafters & Jacks
FIGURE 10-22



The Complete Roof System
***FIGURE*10-23**

To better understand the geometry of the hip rafter, take a look at *Figure 10-24* and *10-25*. I like to focus on the unit roof square because it has the same properties as any roof sharing the same unit rise and unit run. There are four elements in *Figure 10-25*, three triangles sitting on one square: Two triangles with 12″ bases representing the common rafters, and a triangle with the diagonal as its base, representing the hip rafter.

The unit square measures 12″ × 12″. The length of the diagonal in figure*10-24* may be calculated by applying the Pythagorean Theorem.

$$\sqrt{12^2+12^2}=16.9706$$

Figure 10-25 gives us a 3-D view of the unit square and the unit triangles. Notice that all three triangles converge at the top left corner of the square.

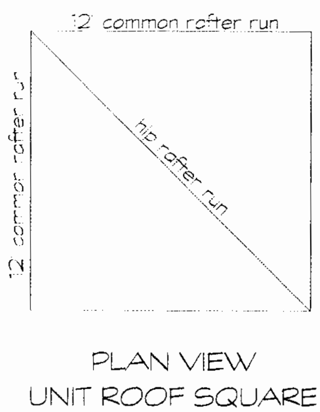


FIGURE 10-24

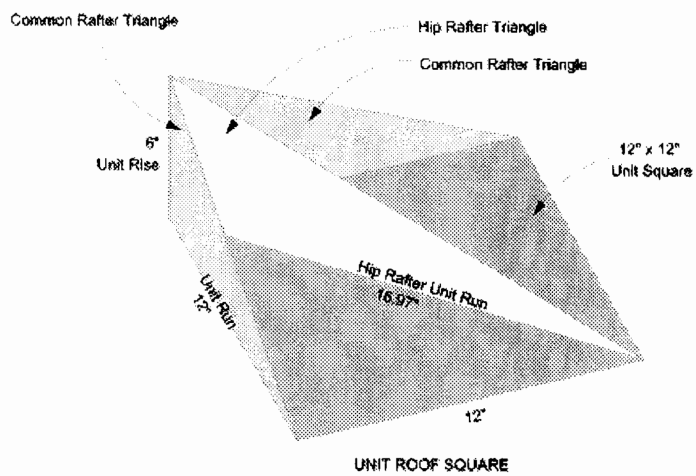


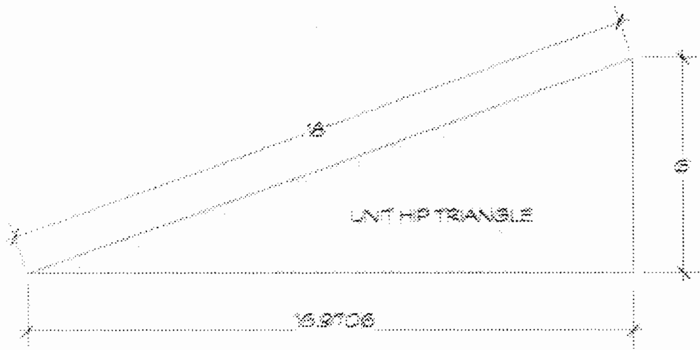
FIGURE 10-25

Armed with the run of the hip triangle and the unit rise of the roof, you can easily calculate the line length of the hip. And, as we did before we can calculate a factor or constant, which when multiplied times the roof run (not the hip run) produces the line length of the hip rafter.

CALCULATING THE HIP CONSTANT

Step 1: Calculate the hypotenuse of the hip triangle. Let's use the hip run that we just calculated and the unit-rise of 6 inches.

$$\sqrt{6^2 + 16.97^2} = 18''$$



The unit line length for a hip with 6/12 slope is 18″

For every 12″ of run the hip rafter line length is 18″

Step 2: Calculate the hip constant by dividing the unit hip line length by the unit run.

$$\frac{18}{12} = 1.5$$

Hip Constant for a 6/12 roof = 1.5

Step 3: Multiply the hip constant times the total run to get the line length of the hip rafter.

Now, simple calculate the hip constants for the other roof slopes and add to your Reference Manual.

HOMEWORK:

Add the hip constants for 2/12 through 14/12 roof slopes to your Reference Manual.

Let's work an actual example:

Figure 10-25 illustrates a plan view of a hip roof with slope of 5/12. To simplify the problem the roof has no roof overhang.

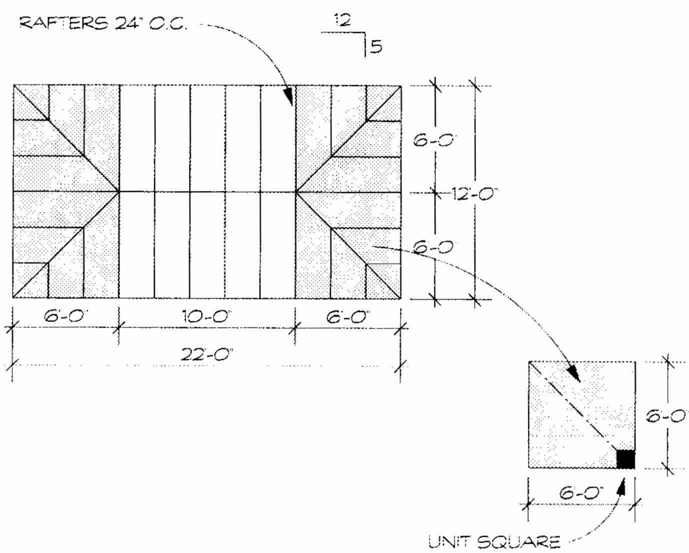


FIGURE 10-25

First lets look at what we need to calculate.

- Ridge Length
- Common Rafter Length
- Hip Rafter Length
- Jack Rafter Length

Step-by-Step Procedure

Step 1: Calculate the ridge length. If each corner of the structure is thought of as a square with sides measuring one-half the span, the ridge measurement in this example would measure 10′- 0″.

Ridge length = Building length - Span

Span = 12′-0″
Total Run 12 ÷ 2 = 6′-0″
Sum of total run at each end = 6′-0″+ 6′-0″ = 12′-0″
Ridge length = 22′-0″ - 12′ 0″ = 10′-0″
Ridge length = 10′- 0″

Step 2: Calculate the rise constant and the line length constant and solve for total rise and the line length of the common rafters. ***(Look them up in your reference manual)***

Rise constant 5/12 = .4167
Total rise 6′-0″ × .4167 = 2′-6″
Line Length Constant 13/12 = 1.0833
Line Length of common rafters 6′-0″ × 1.0833 = 6.72′ = **6′-8 $^5/_8$″**

Step 3: Calculate the hip rafter constant.

$\sqrt{5^2 + 16.97^2} = 17.69$
17.69/12 = 1.4743
Hip Constant = 1.4743

Step 4: Multiply the hip constant times the total run to get the hip rafter line length

1.4743 × 6′-0″ = 8′-10 $^1/_8$″
Line Length of Hip Rafter = 8′- 10 $^1/_8$″

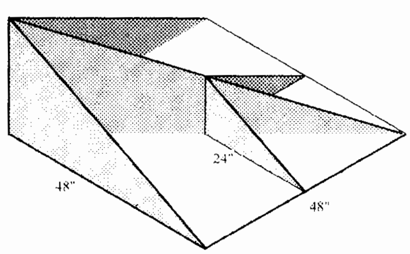
JACK RAFTERS

Step 5: Solve for the lengths of jack rafters

As stated earlier, jack rafters are simply shortened common rafters, shortened because of their intersection with the hip rafter. Look at *Figure 10-26*. As we have studied, rafters are repetitive members usually spaced 16 to 24 inches on-center. Therefore, as a jack rafter shortens, it shortens the same amount each time. The amount the jack rafter is shortened is called the **common difference**.

The common difference measurement is the hypotenuse of a triangle with a base equaling the on-center measurement and an altitude equaling the proportional total rise (rise factor × on-center measurement = total rise). In other words, to find the common difference in jack rafters, you multiply the on-center rafter spacing times the common rafter line length factor.

FIGURE **10-26**



JACK RAFTER TRIANGLES

CALCULATING THE COMMON DIFFERENCE:
Determine the roof rise and the on-center measurement

Unit Rise = 5 inches
On-Center = 24″

Multiply the on-center measurement times the common rafter line length constant.

NOTE: ***If you multiply the constant times 16″ or 24″ OC the product will be in inches. It is easier to convert the on-center measurement to feet before you multiply times the constant.***

2 ×1.0833 = 2′-2″

Common Difference = 2′- 2″

Step 6: Determine the lengths of your jack rafters.
This step is easy because you already have the common difference, all that is necessary now is to subtract it from the common rafter length and you have the first jack rafter. Next you subtract the common difference from the first jack length to get the second jack. You will continue this until you have all the jack rafters for the total run.

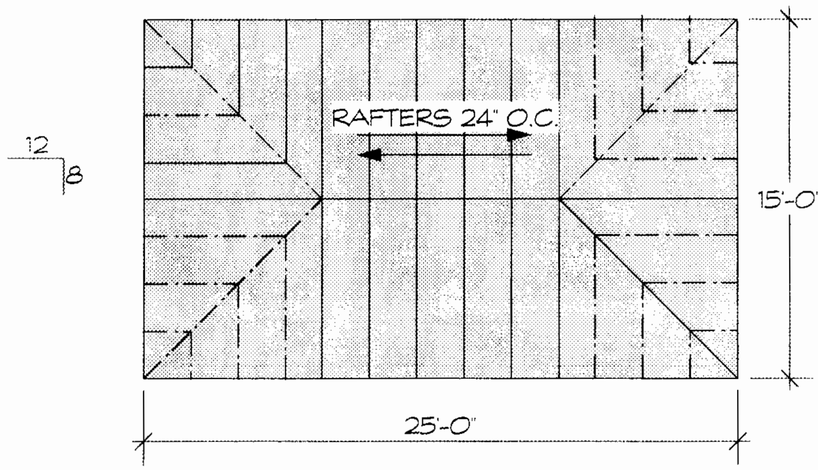
Common Rafter	**6′- 6″**
Common Diff.	**- 2′- 2″**
Jack #1	**4′ - 4 ″**

Jack #1	**4′ - 4 ″**
Common Diff.	**- 2′- 2″**
Jack #2	**2′- 2″**

You have now calculated all of the roof members we set out to solve. It is now your turn to run through the calculations. Use the problem just solved as a model for your calculations.

21. Calculate the following dimensions of the framing members in the roof illustrated below:

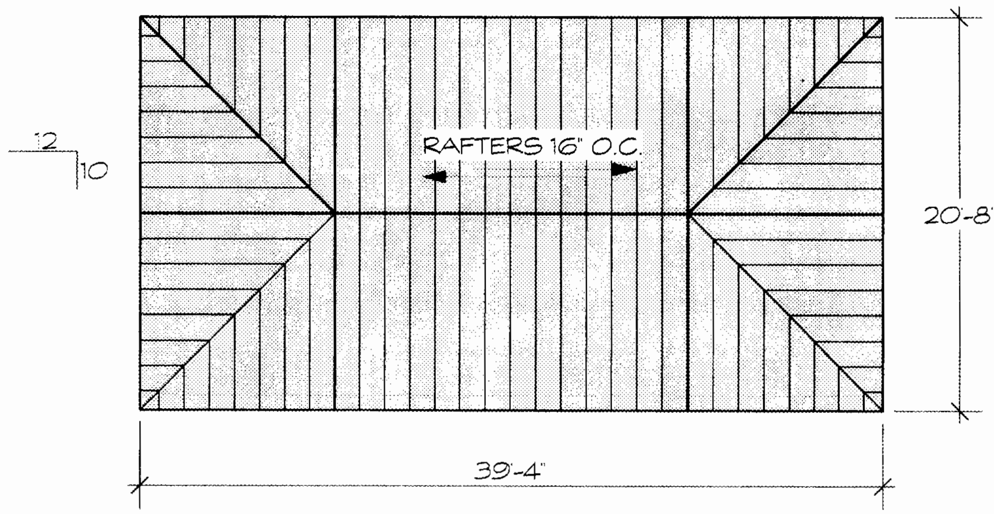
NOTE: ***The number of rafters shown in the illustration is not necessarily the actual number that would be present in a roof with the given dimensions. Your calculations will not be affected by this inconsistency.***



Total run	________
Total rise	________
Ridge length	________
L. L. of common rafters	________
L.L. of hip rafters	________
Common difference of jack rafters	________
Jack 1	________
Jack 2	________
Jack 3	________

22. Calculate the following dimensions of the framing members in the roof illustrated below:

NOTE: ***The illustration below does not necessarily depict the actual number of rafters that would be present in a roof with the given dimensions. Your calculations will not be affected by this inconsistency.***



Total run	________
Total rise	________
Ridge length	________
L. L. of common rafters	________
L.L. of hip rafters	________
Common difference of jack rafters	________
Jack 1	________
Jack 2	________
Jack 3	________
Jack 4	________
Jack 5	________
Jack 6	________
Jack 7	________

ROOF OVERHANG

Up to this point we have been calculating the theoretical rafter lengths measured from the outside of the exterior wall to the centerline of the ridge but the next problem will ask you to calculate the total length of the rafter including the **rafter tail**. As you will remember from *figure 10-15,* the overhang is measured horizontally from the outside of the exterior wall to the end of the common rafter and the tail is simply the extension of the rafter beyond the exterior wall.

There are two ways to approach this problem. The easiest way is to calculate the tail length is to add the overhang to the total run and multiply times the line length factor. Another way is to multiply the overhang times the line length constant and add this product to the line length you calculated for the total run of the building.

Example: 6/12 slope
Overhang = 24″
Total Run = 15′-0″
Common Difference @ 16″ o.c. 1.118 × 1.333 = 1′-5 $^{7}/_{8}$″
LL Constant for Commons= 1.118
LL Constant for Hips = 1.5023

Total run + overhang	15′ + 2′	= **17 ′- 0″**
Total length of commons	17′ × 1.118	= **19′- 0 $^{1}/_{16}$″**
Total length of hips	17 × 1.5023	= **25′-6 $^{1}/_{2}$″**

Total Length of Jacks:

Jack #1	19′-0 $^{1}/_{16}$″ - 1′-5 $^{7}/_{8}$″	= **17′-6 $^{3}/_{16}$″**
Jack #2	17′-6 $^{3}/_{16}$″ - 1′-5 $^{7}/_{8}$″	= **16′- 5″**
Jack #3	16′- 5″ - 1′-5 $^{7}/_{8}$″	= **14′ - 6 $^{7}/_{16}$″**
Jack #4	14′ - 6 $^{7}/_{16}$″ - 1′-5 $^{7}/_{8}$″	= **13'-0 $^{9}/_{16}$"**
Jack #5	13'-0 $^{9}/_{16}$" - 1′-5 $^{7}/_{8}$″	= **11'-6 $^{7}/_{8}$"**
Jack #6	11'-6 $^{7}/_{8}$" - 1′-5 $^{7}/_{8}$″	= **10'-0 $^{3}/_{4}$"**
Jack #7	10'-0 $^{3}/_{4}$" - 1′-5 $^{7}/_{8}$″	= **8'-6 $^{7}/_{8}$"**
Jack #8	8'-6 $^{7}/_{8}$" - 1′-5 $^{7}/_{8}$″	= **7'- 1"**
Jack #9	7'- 1" - 1′-5 $^{7}/_{8}$″	= **5'-7 $^{1}/_{8}$"**
Jack #10	5'-7 $^{1}/_{8}$" - 1′-5 $^{7}/_{8}$″	= **4'- 1 $^{1}/_{4}$"**

Here are two roof-framing scenarios containing all of the problems we have covered in this chapter. Have fun!

23. Situation:
    - Hip roof
    - Span = 19′-5 $^{3}/_{16}$″
    - Slope = 9/12
    - HAP = 3″
    - Roof over hang = 16″
    - Repetitive spacing = 16″ o.c.
    - Common rafter size = 2 X 8
    - Ridge = 2 X 10
    - Hip Rafters = 2 X 10

FILL IN THE BLANKS:

Total run = ____________

Total rise = ____________

L. L. of common rafter ___________ + Tail ________ = ____________

Length of temporary ridge support extending from top plate to bottom of ridge = ______________

L. L. of hip ________ + Tail ________ = ____________

L. L. of jack rafter 1 _________ + Tail ________ = ____________

L. L. of jack rafter 2 _________ + Tail ________ = ____________

L. L. of jack rafter 3 __________+ Tail ________ = ____________

L. L. of jack rafter 4 __________+ Tail ________ = ____________

L. L. of jack rafter 5 __________+ Tail ________ = ____________

L. L. of jack rafter 6 __________+ Tail ________ = ____________

24. Situation:
    - Hip roof
    - Span = 32′-7″
    - Slope = 7/12
    - HAP = 4″
    - Roof over hang = 21″
    - Repetitive spacing = 24″ o.c.
    - Common rafter size = 2 ×10
    - Ridge = 2 × 14
    - Hip Rafters = 2 × 12

FILL IN THE BLANKS:

Total run = ___________

Total rise = ___________

Line length of common rafter ________+ Projection _______ = Total Rafter Length ___________

Length of temporary ridge support extending from top plate to bottom of ridge = ____________

Line length of hip ________ + Tail _______ = ___________

Line length of jack rafter 1 ________ + Tail _______ = ___________

Line Length of jack rafter 2 ________ + Tail _______ = ___________

Line Length of jack rafter 3 ________ + Tail _______ = ___________

Line Length of jack rafter 4 ________ + Tail _______ = ___________

Line Length of jack rafter 5 ________ + Tail _______ = ___________

Line Length of jack rafter 6 ________ + Tail _______ = ___________

I hope that you see the beauty of using constants to calculate various roof components. They are logical and easy to calculate. All you need is the unit rise and the total run or span and you can calculate all the framing members of a roof. If the overhang changes or the span changes all you have to do is multiply the new values times the appropriate constant.

Chapter 10

1. 19′-2 $^1/_2$″
2. 34′-6″
3. 27′-7 $^5/_{16}$″
4. 56′-8 $^3/_8$″
5. No
6. 27′-6 $^3/_8$″
7. 9′-4 $^1/_{16}$″
8. 9′-7$^3/_8$″
9. 60°
10. 75°
11. 96°
12. 1800
13. Label diagram

14. Rise factor=.3333
    Total run=16′, Line
    Length = 16′-10$^3/_8$″
    L.L. factor = 1.0541
    Total rise = 5′-4″

15. Span = 34′-10″
    Rise Factor =.6667
    L.L.=20′-11$^3/_{16}$″ Unit
    L.L. factor = 1.2019
    Total rise=11′-7$^5/_{16}$″

16. Rise factor = .75
    Total run=14′-8″
    Total rise = 11′-0″
    L.L. Factor = 1.25
    L.L. Factor = 1.25
    L.L.=18′-4″

17. 4′-8″
    14′-9$^1/_{16}$″
    1′-7″
    16′-4$^1/_{16}$″
    $^1/_4$″
    4′-1$^1/_2$″

18. 9′-2$^1/_4$″,
    18′-2$^{13}/_{16}$″
    1′-6 $^1/_2$″
    19′-9 $^5/_{16}$″

19. 5′-8 $^3/_{16}$″
    12-8 $^3/_8$″,
    1′-8 $^1/_8$″
    14′-4 $^1/_2$″
    $^3/_8$″
    5′-1 $^9/_{16}$″

20. 11′-3 $^5/_8$″
    14′-8 $^1/_2$″,
    3′- 0 $^7/_{16}$″
    17′-8 $^{15}/_{16}$″,
    $^5/_8$″
    8′-7 $^1/_8$″

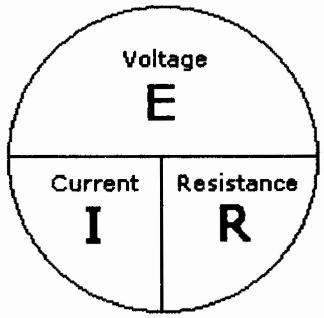
21. 7′-6″, 5′-0″, 10′-0″,
    9-0 $^3/_{16}$″, 11′-8 $^{11}/_{16}$″
    2′-4 $^7/_8$″
    6′-7 $^5/_{16}$″
    4′-2 $^1/_2$″
    1′-9 $^5/_8$″

22. 10'-4"
    8'-7 $^5/_{16}$"
    18'-8"
    13'-5 $^7/_{16}$"
    16'-11 $^9/_{16}$"
    1′-8 $^{13}/_{16}$″
    11'-8 $^9/_{16}$"
    9-11 ¾ "
    8'-2 $^{15}/_{16}$"
    6'-6 $^1/_8$"
    4'-9 $^5/_{16}$"
    3'-0 $^1/_2$"
    1'-3 $^5/_8$"

23. 9′-8 $^9/_{16}$″
    7′-3 $^7/_{16}$″
    12′-1 $^3/_4$″
    1′-8 Com. Proj
    13′-9 $^3/_4$″
    6′-8 $^5/_8$"
    15′-6 $^5/_8$″
    2′-1 $^5/_8$" Hip Proj.
    17′-8 $^1/_4$″
    10′-5 $^3/_4$″ 12′-1 $^3/_4$″
    8′-9 $^3/_4$″ 10′-5 $^3/_4$″
    7′-1 $^3/_4$″ 8′-9 $^3/_4$″
    5′-5 $^3/_4$″ 7′-1 $^3/_4$″
    3′-9 $^3/_4$″ 5′-5 $^3/_4$″
    2′-1 $^3/_4$″ 3′-9 $^3/_4$″

24. 16′-3 $^1/_2$″
    9′-6 $^1/_{16}$″
    Commons=
    18′-10 $^5/_{16}$″
    Proj =.2-0 $^5/_{16}$"
    Total =20′-10 $^5/_8$″
    Ridge Support
    8′- 8 $^3/_8$″
    Hip=
    24′-11 $^1/_{16}$″
    2’-8 $^1/_8$" Proj=
    27′-7 $^3/_{16}$″
    Jacks:
    16′-6 $^9/_{16}$″ 18′-6 $^7/_8$″
    14′-2 $^3/_4$″ 16′-3 $^1/_{16}$″
    11′-11″ 13′-11 $^5/_{16}$″
    9′-7 $^3/_{16}$″ 11′-7 $^1/_2$″
    7′-3 $^7/_{16}$″ 9′-3 $^3/_4$″
    4′-11 $^5/_8$″ 6′-11 $^{15}/_{16}$″

Ohm's Law circle formulas:



I = E/R

E = I x R

R = E/I

Ohm's Law demonstrates the relationship between current (I), voltage (E), and resistance (R) in a direct current, or in an alternating current circuit that supplies only resistive loads. Derived formulas include:

I = E / R **E = I x R** **R = E / I**

Ohm's Law states that:

Current is directly proportional to voltage. If the voltage is increased by a given percentage, current increases by that same percentage. If the voltage is decreased by a given percentage, current decreases by the same percentage.

Current is inversely proportional to resistance. An increase in resistance results in a decrease in current. A decrease in resistance results in an increase in current.

Opposition to current flow:

In a direct current circuit, the physical resistance of the conductor opposes the flow of electrons. In an alternating current circuit, three factors oppose current flow. They are pure resistance, inductive reactance, and capacitive reactance. The opposition to current flow, due to a combination of resistances and reactance, is called impedance, measured in ohms.

Pure resistance: The opposition to the flow of an electrical current in the conductors.

Inductive reactance: The resistance to the flow of an alternating current produced by an electromagnetic induction.

Capacitive reactance: The resistance to the flow of an alternating current produced by a capacitor in the circuit.

Examples:

Ampere:

A 120-volt power source supplies a lamp with a resistance of 192 ohms. What is the current flow of the circuit?

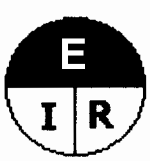
1. *What are you looking for?* What is the current (I)?
2. *What do you know?* E = 120 volts, R = 192 ohms.
3. *The formula is:* I = E/R
4. I = 120 volts / 192 ohms = **0.625 amps**

I = E/R

Voltage:

What is the voltage drop of two #12 conductors that supply a 16 ampere load located 50 feet from the power supply? The total resistance of both conductors is 0.2 ohms.

1. *What are you looking for?* What is the voltage (E)?
2. *What do you know?* I = 16 amperes, R = 0.2 ohms
3. *The formula is:* E = I x R
4. E = 16 amperes x 0.2 ohms = **3.2 volts**



E = I x R

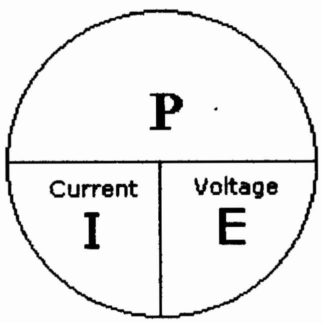
Resistance:

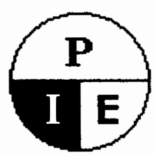
What is the resistance of the circuit conductors when the conductor voltage drop is 3 volts and the current flowing through the conductors is 100 amperes?

1. *What are you looking for?* What is the resistance (R)?
2. *What do you know?* E = 3 volts dropped, I = 100 amperes
3. *The formula is:* R = E/I
4. R = 3 volts / 100 amperes = **0.03 ohms**

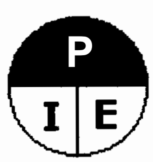
R = E/I

Power circle formulas:

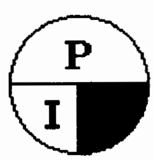




I = P/E



P = I x E



E = P/I

The power circle formula shows the relationships between power (P), current (I), and voltage (R) or PIE. Derived formulas include:

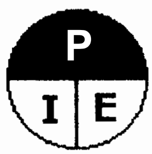
I = P / E **P = I x R** **E = P / I**

Examples:

Power:

What is the power loss, in watts, for two conductors that carry 12 amperes and have a voltage drop of 3.6 volts?

1. *What are you looking for?* What is the power (P)?
2. *What do you know?* E + 3.6 volts dropped, I = 12 amperes
3. *The formula is:* P = E x I
4. P = 3.6 volts x 12 amperes = **43.2 watts**

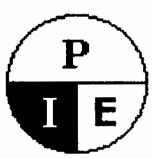


P = I x E

Current:

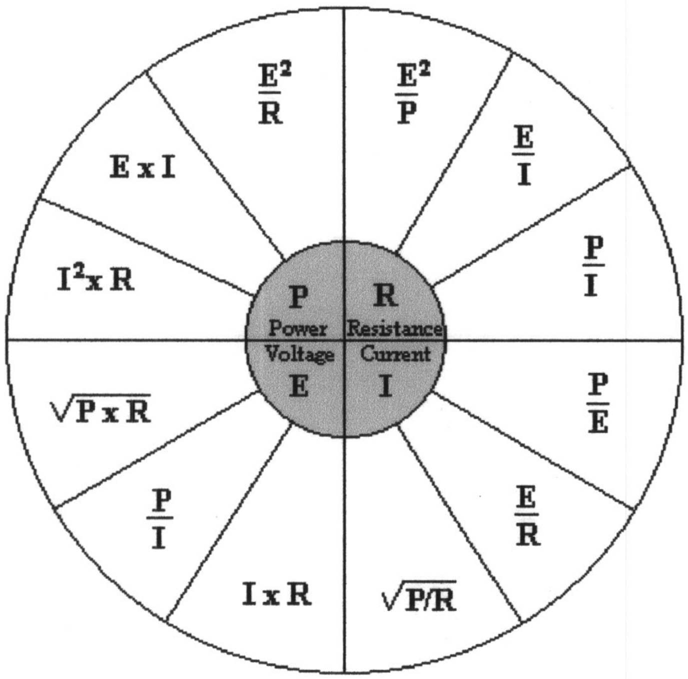
What is the current flow, in amperes, in the circuit conductors that supply a 7.5 kW heat strip rated at 240 volts when connected to a 240-volt power supply?

1. *What are you looking for?* What is the current (I)?
2. *What do you know?* P = 7,500 watts, E = 240 volts
3. *The formula is:* I = P/E
4. I = 7,500 watts / 240 volts = **31.25 amperes**



I = P/E

Formula Wheel

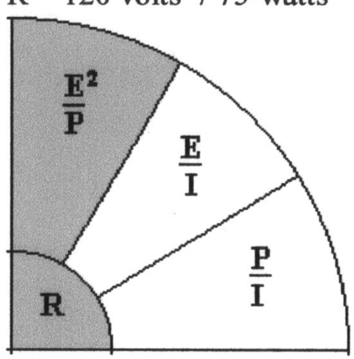


The formula wheel combines Ohm's Law and the power formulas. The formula wheel is divided up into four sections with three formulas in each section for a total of twelve formulas.

Examples:

Resistance:

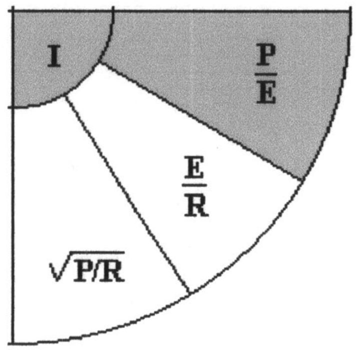
What is the resistance of a 75-watt light bulb rated at 120 volts?
$R = E^2 / P$
E = 120 volts, P = 75-watt rating
$R = 120$ volts^2 / 75-watts = **192 ohms**



Current:

What is the current flow of a 10 kW heat strip connected to a 230 volt, single phase, power supply?

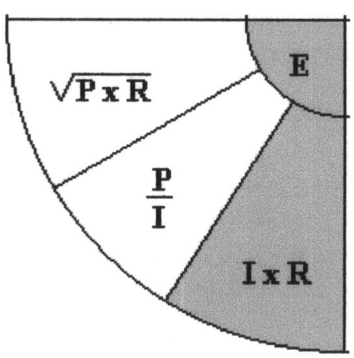
I = P / E
P = 10,000 watts, E = 230 volts
I = 10,000 watts / 230 volts = **43 amperes**



Voltage:

What is the voltage drop of 200 feet of #12 conductor that carries 16 amperes? The resistance of #12 copper conductor is 2 ohms per 1,000 feet.

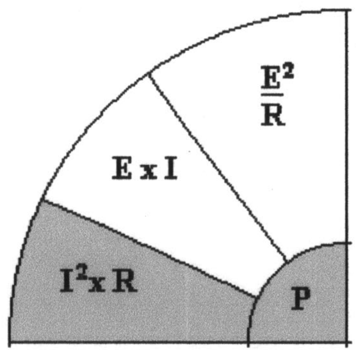
E = I x R
I = 16 amperes, R = 2 ohms / 1,000 = 0.002 ohms per foot x 200 feet = 0.4 ohms
E = 16 amperes x 0.4 ohms = **6.4 volts dropped**



Power:

The total resistance of two #12 copper conductors, 75 feet long, is 0.3 ohm (0.15 ohms for each conductor). The current of the circuit is 16 amperes. What is the power loss of the conductors, in watts per hour?

$P = I^2 \times R$
$I = 16$ amperes, $R = 0.3$ ohms
$P = 16 \text{ amperes}^2 \times 0.3 \text{ ohms} = 76.8$ watts per hour



Power changes with the square of the voltage

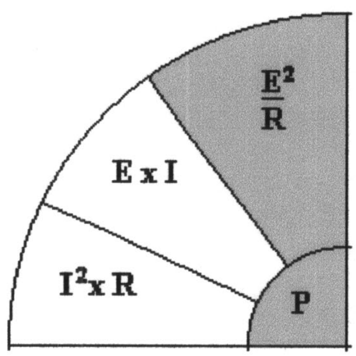
The power consumed by a resistor is affected by the voltage applied. Power is proportional to the square of the voltage and directly proportional to the resistance.

Example:

Power Changes With the Square of the Voltage:

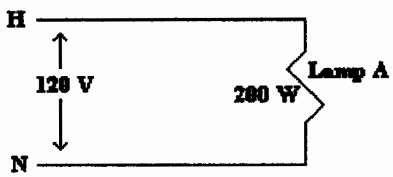
What is the power consumed of a 9.6 kW heat strip rated at 230 volts connected to 115-, 230- and 460-volt power supplies? The resistance of the heat strip is 5.51 ohms.

$P = E^2 / R$
E = 115 volts, 230 volts and 460 volts, R = 5.51 ohms
P = 115 volts^2 / 5.51 ohms = **2,400 watts**
P = 230 volts^2 / 5.51 ohms = **9,600 watts**
P = 460 volts^2 / 5.51 ohms = **38,403 watts**

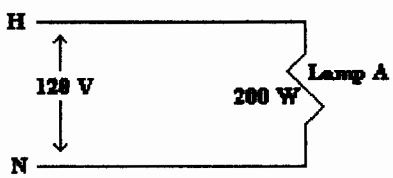


Name: ____________
Date: ____________

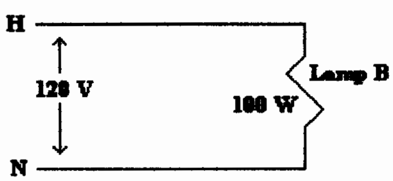
1. Solve for the ohms resistance of Lamp A.



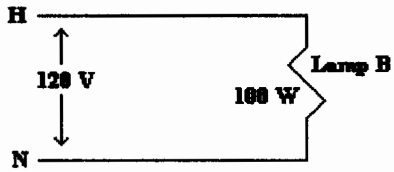
2. Solve for the current of Lamp A.



3. Solve for the ohms resistance of Lamp B.



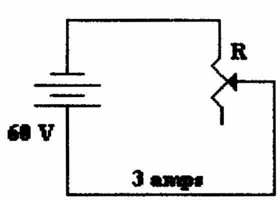
4. Solve for the current of Lamp A.



5. How much current would an applied voltage of 10 volts cause through a resistance of 5 ohms?

6. A 20-ohm resistor used as the load in a circuit having a 100-volt battery as a voltage source. What is the current rating for the circuit?

7. In the following circuit diagram, 3 amperes of current flows in the circuit when the rheostat is set at the middle of its range. How much resistance is being added to the circuit?



8. What is the resistance in ohms of an electric resistance heater rated at 1500 watts and operating at 120 volts? What is the current draw?

9. You have a tenant that has an electric heating element used for heating PVC conduit for bending. It is rated at 2250 watts and operating on 120 volts. It is located 300 feet from the circuit breaker panel using #12 copper conductor. The resistance of #12 copper conductor is 2 ohms per 1,000 feet. It is tripping a 20-amp circuit breaker. What is the most likely problem?

a. Heater is shorted out or otherwise defective.
b. #12 copper conductor is too small.
c. The circuit breaker is defective.
d. The circuit beaker is rated to small.

10.

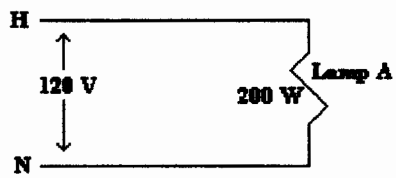
a. Doubling the resistance of a circuit has what effect on the current, if the source voltage is held constant?

b. Halving the source voltage has what effect on the current, if the resistance is held constant?

c. Doubling both the source voltage and the resistance has what effect on the current?

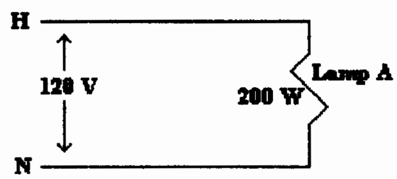
Answers:

1. Solve for the ohms resistance of Lamp A.



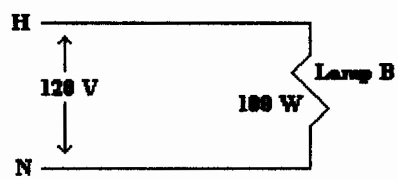
R = E^2 / W = 120 X 120 / 200 = 72 ohms

2. Solve for the current of Lamp A.



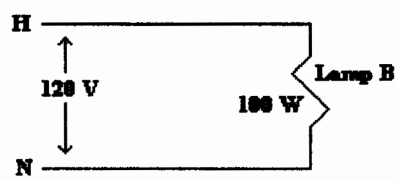
I = W / E = 200 / 120 = 1.6666 amps

3. Solve for the ohms resistance of Lamp B.



R = E2 / W = 120 X 120 / 100 = 144 ohms

4. Solve for the current of Lamp A.



I = W / E = 100 / 120 = .8333 amps

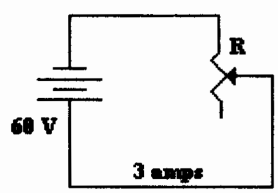
5. How much current would an applied voltage of 10 volts cause through a resistance of 5 ohms?

I = E / R = 10 / 5 = 2 amps

6. A 20-ohm resistor used as the load in a circuit having a 100-volt battery as a voltage source. What is the current rating for the circuit?

I = E / R = 100 / 20 = 5 amps

7. In the following circuit diagram, 3 amperes of current flows in the circuit when the rheostat is set at the middle of its range. How much resistance is being added to the circuit?



R = E / I = 60 / 3 = 20 ohms

8. What is the resistance in ohms of an electric resistance heater rated at 1500 watts and operating at 120 volts? What is the current draw?

R = E^2 / P = 120^2 / 1500 = 9.6 ohms
I = P / E = 1500 / 120 = 12.5 amps

9. You have a tenant that has an electric heating element used for heating PVC conduit for bending. It is rated at 2250 watts and operating on 120 volts. It is located 300 feet from the circuit breaker panel using #12 copper conductor. The resistance of #12 copper conductor is 2 ohms per 1,000 feet. It is tripping a 20-amp circuit breaker. What is the most likely problem?

e. Heater is shorted out or otherwise defective.
f. #12 copper conductor is too small.
g. The circuit breaker is defective.
h. The circuit beaker is rated to small.

The rated amps of the heater:
I = P / E = 2250 / 120 = 18.75 amps
Voltage drop in the conductor:
E = I x R = 18.75 x ((.2 / 1000) x 300) = 18.75 x .6 = 11.25 volts drop

Actual amp draw of the heater:

I = P / E = 2250 / (120 – 11.25) = 2250 / 108.75 = 20.689 amps

<u>The circuit beaker is rated to small.</u>

10.

d. Doubling the resistance of a circuit has what effect on the current, if the source voltage is held constant?

e. Halving the source voltage has what effect on the current, if the resistance is held constant?

f. Doubling both the source voltage and the resistance has what effect on the current?

a. The current doubles.
b. The current doubles.
c. Stays the same.